MOE 2017

53) find a point of the straight line $\frac{x}{3} = \frac{y+1}{1} = \frac{z-3}{2}$ such that its x-coordinate equals double its y – coordinate

(a) (- 6, - 3, - 1) (b) (4, 2, - 1) (c) (6, 3, - 1) (d) (2, 1, - 1)

Complete the following:

S.B.2017

54) the measure of the angle between the two straight lines 2 x= 3 y = - z and 6 x = - y = - 4 z equals ……..

S.B.2017

55) the length of the perpendicular drawn from the point (- 1, 0, 1) to the straight line $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z+1}{-1}$ equals …….

S.B.2017

56) the parametric equation for the straight line passing through the two points A(- 1, 0, 3) and B(1, - 1, 0) are …..

S.B.2017

57) the equation of the straight line passing through the two points A (2, - 1, 4) and B (- 1, 0, 2) is …..

S.B.2017

58) if the straight line $\frac{x+3}{2} = \frac{y+1}{-6} = \frac{z-2}{K}$ is parallel to the straight line $\frac{x+2}{4} = \frac{y-5}{m} = \frac{z-1}{3}$, then k + m = …..

S.B.2017

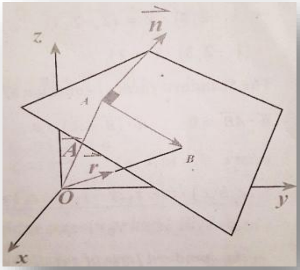
59) the direction vector of the straight line $\frac{x+2}{3} = \frac{z-1}{2}$ equals ……

**** Vector from of equation of the plane in space :***

If the point A (x_1 , y_1, z_1) lies in the plane and its position vector is $\vec{A}$ and the vector $\vec{n}$ where $\vec{n}$ = (a, b, c) is normal direction

Vector to the plane and the point

B (x, y, z) is appoint on the plane its position $\vec{r}$ then :



$\vec{n} \, . \, \overrightarrow{AB} = 0$, $\vec{n} \, . \, (\vec{B} - \vec{A}) = 0$

$\vec{n} \, . \, \vec{B} - \vec{n} \, . \, \vec{A} = 0$

$\vec{n} \, . \, \vec{B} = \vec{n} \, . \, \vec{A} \quad \because \vec{r} = \vec{B}$

$\rightarrow \vec{n} \, . \, \vec{r} = \vec{n} \, . \, \vec{A}$ $\rightarrow$ the vector form of equation of plane in space

To find the vector equation of plane in space you have to know the point in the plane and direction vector perpendicular to the plane .

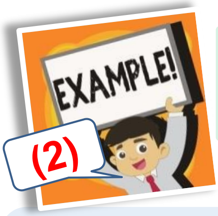
find the vector form of equation plane passing the point (0, 1, 1) and the vector $\vec{n} = \hat{\imath} + \hat{\jmath} + \hat{k}$ is perpendicular to the plane

SOLUTION

$\vec{n} = (1, 1, 1)$, $\vec{A} = (0, 1, 1)$ $\because \vec{n} \, . \, \vec{r} = \vec{n} \, . \, \vec{A}$

$\rightarrow (1, 1, 1) \, . \, \vec{r} = (1, 1, 1) \, . \, (0, 1, 1) \rightarrow (1, 1, 1) \, . \, \vec{r} = 0 + 1 + 1$

$\rightarrow (1, 1, 1) \, . \, \vec{r} = 2$



find the vector form of the equation of the plane passing the point (3, - 3, 1) and the vector $\vec{n}$ = (1, - 2, 3) is perpendicular to the plane

SOLUTION

$\vec{n} = (1, -2, 3)$, $\vec{A} = (2, -3, 1)$ $\because \vec{n} \, . \, \vec{r} = \vec{n} \, . \, \vec{A}$

$\rightarrow (1, -2, 3) \, . \, \vec{r} = (1, -2, 3) \, . \, (2, -3, 1) \rightarrow (1, -2, 3) \, . \, \vec{r} = 2 + 3 + 6$

$\rightarrow (1, -2, 3) \, . \, \vec{r} = 11$

The standard form of equation of plane in space:

$\vec{n} \, . \, \vec{A}B = 0$, $\vec{n} \, . \, (\vec{B} - \vec{A}) = 0$, $\vec{n} \, . \, (\vec{n} - \vec{A}) = 0$

Where $\vec{n}$ = (a, b, c) , $\vec{r}$ = (x, y, z) , $\vec{A} = (x_1, y_1, z_1)$

$\therefore$ (a, b, c) . ($x - x_1, y - y_1, z - z_1$) = 0

$\therefore a (x - x_1) + b (y - y_1) + c (z - z_1) = 0$

$\rightarrow$ The standard form of equation of plane in space:

The general form of equation of plane in space:

$\because a (x - x_1) + b (y - y_1) + c (z - z_1) = 0$

$\therefore ax + by + cz + (- ax_1 - by_1 - cz_1) = 0$

Let $- ax_1 - by_1 - cz_1 = d$

$\therefore ax + by + cz + d = 0$

The general form of equation of plane in space

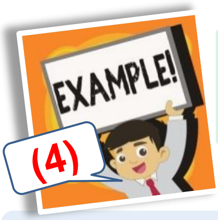
Find the standard form and general form of equation of the plane passing the point (3, - 5, 2) and the vector $\vec{n}$ = (2, 1, 1)

SOLUTION

The standard form: $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

$\rightarrow 2(x - 3) + 1(y + 5) + 1(z - 2) = 0$

The general form: $2x - 6 + y + 5 + z - 2 = 0 \rightarrow 2x + y + z - 3 = 0$



Find the different forms of equation of plane passing the point (- 3, 4, 2) and the vector $\vec{n}$ = (1, - 1 , 3) is perpendicular to the plane

SOLUTION

The vector form : $\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{A}$

$(1, -1, 3) \cdot \vec{r} = (1, -1, 3) \cdot (-3, 4, 2) = -3 - 4 + 6$

$(1, -1, 3) \cdot \vec{r} = -1$

The standard form : $1(x+3) - 1(y - 4) + 3(z - 2) = 0$

$(x + 3) - (y - 4) + 3(z - 2) = 0$

The general form : $x + 3 - y + 4 + 3z - 6 = 0$

$x - y + 3z + 1 = 0$

The equation of the plane passing through three non-collinear points

find the different forms of equation of plane passing through the point (3, - 1 , 0) , (2, 1, 4) , (0, 3, 3)

SOLUTION

First , we must make sure that the points are non-collinear

Let A = (3, -1, 0) , B = (2, 1, 4), C = (0, 3, 3)

$\overrightarrow{AB} = \vec{B} - \vec{A} = (-1, 2, 4)$ $\overrightarrow{AC} = \vec{C} - \vec{A} = (-3, 4, 3)$

$\frac{a_1}{a_2} = \frac{1}{-3} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}$ $\because \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\therefore$ the three points non collinear

To find a direction vector $\vec{n}$ perpendicular to plane :

$$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} \quad \vec{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 2 & 4 \\ -3 & 4 & 3 \end{vmatrix} \rightarrow \vec{n} = -10\,\hat{\imath} - 9\,\hat{\jmath} + 2\,\hat{k}$$

$\rightarrow \vec{n} = (-10, -9, 2)$

The vector form : $\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{A}$

$(-10, -9, 2) \cdot \vec{r} = \vec{n} \cdot \vec{A} \longrightarrow (-10, -9, 2) \cdot (3, -1, 0)$

$(-10, -9, 2) \cdot \vec{r} = -30 + 9 + 0 \longrightarrow (-10, -9, 2) \cdot \vec{r} = -21$

The standard form : $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

$-10(x - 3) - 9(y + 1) + 2(z - 0) = 0 \longrightarrow -10(x - 3) - 9(y + 1) + 2(z) = 0$

The general form $-10x - 9y + 2z + 21 = 0$

Or $(-10, -9, 2) \cdot \vec{r} = -21$

$(-10, -9, 2) \cdot (x,y,z) + 21 = 0 \longrightarrow -10x - 9y + 2z + 21 = 0$

(6) Find the different forms of equation of plane passing through the point (1, 0,0) , (0, 2, 0), (0, 0, 3)

SOLUTION

Let A (1,0, 0) , B(0, 2, 0) , C(0, 0, 3)

$\overrightarrow{AB} = \vec{B} - \vec{A} = (-1, 2, 0)$ $\qquad$ $\overrightarrow{AC} = \vec{C} - \vec{A} = (-1, 0, 3)$

Let the direction vector perpendicular to the plane is $\vec{n}$

$$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} \longrightarrow \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix} \longrightarrow \vec{n} = 6\,\hat{\imath} + 3\,\hat{\jmath} + 2\,\hat{k} = (6, 3, 2)$$

The vector form : $\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{A}$

$(6, 3, 2) \cdot \vec{r} = (6, 3, 2) \cdot (1,0,0) = 0 + 6 \longrightarrow (6, 3, 2) \cdot \vec{n} = 6$

The standard form : $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

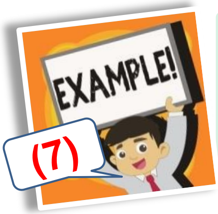
$6(x - 1) + 3(y - 0) + 2(z - 0) = 0 \longrightarrow 6(x - 1) + 3y + 2z = 0$

The general form : $(6, 3, 2) \cdot (x, y, z) = 6 \longrightarrow 6x + 3y + 2z - 6 = 0$

Another solution:

Equation of plane passes the point $(x_1,y_1,z_1),(x_2,y_2,z_2),(x_3,y_3,z_3)$

$$\longrightarrow \begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix}$$



Prove that the two lines $\overrightarrow{r_1} = (3\hat{i} + \hat{j} - \hat{k}) + t_1 (\hat{i} + 2\hat{j} + 3\hat{k})$

$\overrightarrow{r_2} = (2\hat{i} + 5\hat{j}) + t_2 (\hat{i} - \hat{j} + \hat{k})$ are intersecting and find the equation of the plane containing them.

SOLUTION

If the two lines are intersecting then r1 = r2

$(3,1,-1) + t_1(1,2,3) = (2,5,0) + t_2 (1,-1,1)$

$3 + t_1 = 2 + t_2 \quad \rightarrow \quad t_1 - t_2 = -1 \qquad (1)$

$1 + 2t_1 = 5 - t_2 \quad \rightarrow 2t_1 + t_2 = 4 \qquad (2)$

$-1 + 3t_1 = 0 + t_2 \quad \rightarrow 3t_1 - t_2 = 1 \qquad (3)$

From (1) & (2) $t_1 = 1$, $t_2 = 2$ by substitution at (3) L.H.S = R.H.S

∴ the two lines intersect

the direction vector ñ that is perpendicular to the plane

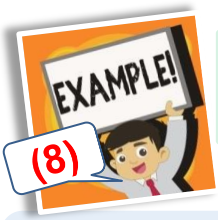
$$\vec{n} = \overrightarrow{d_1} \times \overrightarrow{d_2} \quad \rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{vmatrix} = 5\hat{i} + 2\hat{j} - 3\hat{k} = (5, 2, -3)$$

The vector equation : $\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{A}$

$\rightarrow (5,2,-3) \cdot \vec{r} = (5,2,-3). (3,1,-1) = 15 + 2 + 3$

$\rightarrow (5,2,-3) \cdot \vec{r} = 20$

the general form: $(5,2,-3) . (x,y,z) = 20 \rightarrow 5x + 2y \rightarrow 3z - 20 = 0$



Prove that the two lines $L_1: 2x = 3y = 4z$, $L_2 : 3x = 2y = 5z$ intersect then find the equation of the plane containing them.

SOLUTION

L1 : $2x = 3y = 4z = t_1 \quad \therefore x = \frac{t_1}{2}, \quad y = \frac{t_1}{3}, \quad z = \frac{t_1}{4}$

equation of $L_1 : \overrightarrow{r_1} = (0,0,0) + t_1 (6,4,3)$

L2 : $3x = 2y = 5z = t2 \quad \therefore x = \frac{t_2}{3}, \quad y = \frac{t_2}{2}, \quad z = \frac{t_2}{5}$

equation of $L_2 : \overrightarrow{r_2} = (0,0,0) + t_2(10,15,6)$

if the two lines intersect: $\therefore \overrightarrow{r_1} = \overrightarrow{r_2} \quad \therefore t_1 (6,4,3) = t_2 (10, 15,6)$

$6t_1 = 10t_2$ 1 $t_2 : \frac{3}{5} t_1$ (1) , $\therefore 4t_1 = 15t_2$ (2) , $3 t_1 = 6t_2 \quad \therefore t_1 = 2t_2$ (3)

From (1) & (2), $t_1 = 0$ & $t_2 = 0$ ∵ $t_1 = 0$& $t_2 = 0$ satisfy (3)

∴ The two lines intersect o (0,0,0) lie on the plane, let $\vec{n}$ be normal vector to plane

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 4 & 3 \\ 10 & 15 & 6 \end{vmatrix} = (-21, -6, 50) \therefore \text{eq.of plane is } -21x - 6y + 50z = 0$$

EXAMPLE! (9)

Find the point of intersection of the line 2x = 3y - 1 = z - 4 with the plane 3x + y - 2z = 5

SOLUTION

The equation of the plane : y = 5 - 3x + 2z

by substitution in equation of line : 2x = 3(5 - 3x + 2z) - 1 = z - 4

2x = 15 + 6z - 9x – 1 = z – 4 ⟶ 2 x = 14 + 6z - 9x = z - 4

11x – 6z = 14 ⟶ (1) 5z - 9x = -18 ⟶ (2)

by solving (1),(2) x = -38 , z = -72

by substitution in equation of the plane ⟶ y = -25

∴ the intersection point (-38, -25, -72)

EXAMPLE! (10)

Find the intersection point of the line $\vec{r}$ = (1,4,2) + t (3,2,2) with the plane (3,2,2) . $\vec{r}$ = -2

SOLUTION

The equation of plane: 3x + 2y + 2z + 2 = 0

The equation of line: x = 1 + 3t , y = 4 + 2t , z = 2 + 2t

By substitution in equation of plane:

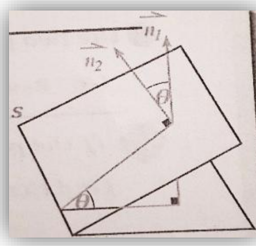
3(1 + 3t) + 2(4 + 2t) + 2(2 + 2t) + 2 = 0 ⟶ 3 + 9t + 8 + 4t + 4 + 4t + 2 = 0

∴ t = -1 ∴ x = -2 , y = 2 , z = 0 ∴ (-2,2,0) is the intersection point

The angle between two planes :

is the measure of the angle between the two direction vectors perpendicular the two plane is given by $\cos\theta = \dfrac{|\overrightarrow{n_1} \cdot \overrightarrow{n_2}|}{\|\overrightarrow{n_1}\|\,\|\overrightarrow{n_2}\|}$

where $0 \le \theta \le 90°$



EXAMPLE! (11)

Find the measure of the angle between the two planes

P_1: (2, -1 , 4) . $\vec{r}$ = 5 , P_2 : 3x – y + 2z = 4

SOLUTION

The direction vector perpendicular to the first plane is n1 = (2, -1, 4) the direction vector vector perpendicular to the second plane is $\overrightarrow{n_2}$ = (3,-1,2) ∴ the measure of the angle between two plane is $\cos\theta = \dfrac{|\overrightarrow{n_1} \cdot \overrightarrow{n_2}|}{\|\overrightarrow{n_1}\|\,\|\overrightarrow{n_2}\|}$ ⟶ $\cos\theta = \dfrac{|(2,-1,4).(3,-1,2)|}{\sqrt{4+16+1}.\sqrt{9+1+4}}$ =

$\dfrac{|6+1+8|}{\sqrt{21}.\sqrt{14}} = \dfrac{15}{7\sqrt{6}}$ ⟶ θ = 28° 58'

EXAMPLE! (12)

Find the measure of the angle between the two planes

x -3y + 2z = 0 , 2x + y -2 = 3

SOLUTION

$\overrightarrow{n_1}$ = (1, - 3, 2) , $\overrightarrow{n_2}$ = (2, 1, - 1), $\cos\theta = \frac{|\overrightarrow{n_1} \cdot \overrightarrow{n_2}|}{\|\overrightarrow{n_1}\| \, \|\overrightarrow{n_2}\|} = \frac{|2-3-2|}{\sqrt{14}\ \sqrt{6}}$

$= \frac{3}{2\sqrt{21}}$ θ = 70°53'36"

Parallel planes and perpendicular planes :

If $\overrightarrow{n_1}$. $\overrightarrow{n_2}$ two direction vectors perpendicular to planes then :

1) the two planes are parallel if $\overrightarrow{n_1} \,//\, \overrightarrow{n_2}$ $\left(ie\ \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}\right)$

2) the two planes aree perpendicular if $\overrightarrow{n_1} \cdot \overrightarrow{n_2} = 0$

(ie. a_1, a_2, + b_1 b_2 + c_1c_2 = 0)

EXAMPLE! (13)

is the plane 2x - y + kz = 5 is parallel to the plane x + *l* y + 4z = 1 find the value of k , *l*

SOLUTION

∵ the two planes are parallel $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ $\rightarrow \frac{2}{1} = -\frac{1}{l} = \frac{k}{4}$

l = $-\frac{1}{2}$, k = $-\frac{4}{-\frac{1}{2}}$ = 8

EXAMPLE! (14)

If the plane x - 3y + z = 4 is perpendicular to the plane ax +2y + 3z = 2 find the value of a.

SOLUTION

$\overrightarrow{n_1}$ = (1, -3, 1) , $\overrightarrow{n_2}$ = (a, 2, 3) ∵ $P_1 \perp P_2$ ∴ $\overrightarrow{n_1} \cdot \overrightarrow{n_2} = 0$

(1, 3, 1) (a, 2, 3) = 0 → a - 6 + 3 = 0 ∴ a = 3

Find the equation of intersection line of the two planes

x + 2y - 2z = 1 , 2x + y - 3y = 5

SOLUTION

by eliminating x from the two equation :

x + 2y – 2z = 1 → (1) 2x + y - 3y = 5 → (2)

multiplying (1) (by - 2)

-2x - 4y + 4z = -2 → - 3y + z = 3

2x + y - 3z = 5 → z = 3 + 3y

by eliminating y from the two equations

x + 2y - 2z = 1

- 4x - 2y + 6z = -10 → -3x+ 4z = -9

z = $\frac{3x-9}{4}$ → the equation of line of intersection

$\frac{3x-9}{4} = \frac{3y+9}{1} = \frac{z}{1}$

SOLUTION

∵ -3y + z = 3 let z = k - 3y + k = 3 y = $\frac{k-3}{3}$

-3x + 4z = -9 ∴ x = $\frac{9+4k}{3}$ x = $\frac{9+4k}{3}$ = 3 + $\frac{4}{3}$ k

the parametric equation of intersection line is

x = 3 + $\frac{4}{3}$ k , y = -1 + $\frac{1}{3}$ k , z = k

SOLUTION

The intersection line is perpendicular to the two vectors $\overrightarrow{n_1}$, $\overrightarrow{n_2}$ which are perpendicular to the two planes.

∴ the direction vector of the intersection line $\vec{d}$ is given by

$\vec{d} = \overrightarrow{n_1} \times \overrightarrow{n_2}$

$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & -2 \\ 2 & 1 & -3 \end{vmatrix}$ = - 4î - j - 3$\hat{k}$ to find a point on the intersection line let x = 1

by substitution on the equation of the first plane 2y - 2z = 0 → (1) by substitution on the equation of the second plane y- 3z = 3 → (2)

by solving the two equation : z = - $\frac{3}{2}$, y = - $\frac{3}{2}$

the point $\left(1, -\frac{3}{2}, -\frac{3}{2}\right)$ lies on the line of intersection

∴ The equation of line of intersection is

$\vec{r} = \left(1, -\frac{3}{2}, -\frac{3}{2}\right)$ + t (- 4, -1, -3)

Find the equation of the line of intersection of the two planes

$3x - y + 2z = 3$, $x - 2y + 5z = 2$

SOLUTION

By eliminating y from the equations by multiplying the first equation by (-2) and adding the two equations

$-6x + 2y - 4z = -6,\ x - 2y + 5z = 2 \rightarrow -5x + z = -4 \therefore z = 5x - 4$

By eliminating x from the equations by multiplying the second equation by (-3) and adding the two equations

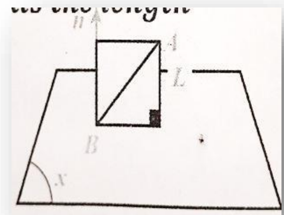
$-3x + 6y - 15z = -6$, $3x - y + 2z = 3 \rightarrow 5y - 13z = -3 \therefore z = \frac{5y + 3}{13}$

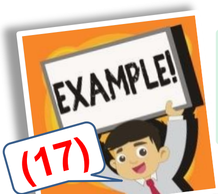
The equation of the line of the intersection: $z = 5x - 4 = \frac{5y + 3}{13}$

The length of the perpendicular from a point to a plane :

If $A(x_1, y_1, z_1)$ is a point out of the plane (x) and the point B belongs to the plane (x) $\vec{n}$ is a direction vector perpendicular to the plane then the distance between the point A and the plane equals the length of projection of $\overrightarrow{BA}$ on $\vec{n}$ is

$$L = \frac{|\overrightarrow{AB} \cdot \vec{n}|}{\|\vec{n}\|}$$





Find the length of the perpendicular drawn from the point (1, -1, 3) to the plane of equation $\vec{r} \cdot (2 , 2 , -1) = 5$

SOLUTION

$\vec{r} \cdot (2 , 2 , -1) = 5 \qquad \therefore \vec{n} = (2,2,-1)$

let the plane intersect z - axis at the point (0,0,z)

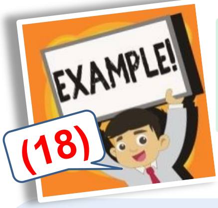
$\therefore (0 , 0 , z) (2 , 2 , -1) = 5 \quad \rightarrow z = -5$

$\therefore$ the point B (0,0,-5) *l*ying on the plane.

$\overrightarrow{BA} = \vec{A} - \vec{B} \qquad \overrightarrow{BA} = (1, -1, 3) - (0, 0, -5) \qquad \overrightarrow{BA} = (1, -1, 8)$

the length of the perpendicular $L = \frac{|\overrightarrow{BA} \cdot \vec{t}|}{\|\vec{t}\|}$

$= \frac{|(1,-1,8) \cdot (2,2,-1)|}{\sqrt{2^2+2^2+(-1)^2}} = \frac{8}{3}$ unit



Find the perpendicular length drawn from the point (-2,1,4) to the plane whose equation $\overrightarrow{r}$. (1,-3,2) = 4

SOLUTION

$\because \overrightarrow{r}$ (1,-3,2) = 4 $\quad \therefore \overrightarrow{n}$ = (1, -3,2)

let the plane intersect z - axis at the point B(0,0,z) →

$\overrightarrow{r}$. (1-3,2) = 4

(0,0,z) . (1, -3, 2) = 4 → 2z = 4 → z = 2

∵ the point B = (0,0,2) r = (0,0,2)

$\overrightarrow{BA} = \vec{A} - \vec{B}$ $\quad$ A = (-2,1,4) $\quad$ $\overrightarrow{BA}$ = (-2,1,4) – (0,0,2)

$\overrightarrow{BA}$ = (-2,1,2)

The length of the perpendicular distance (L)

$$L = \frac{|\overrightarrow{BA} \cdot \vec{n}|}{\|\vec{n}\|} = \frac{|(-2,1,2) \cdot (1,-3,2)|}{\sqrt{1+9+4}} = \frac{|-2-3+4|}{\sqrt{14}} = \frac{1}{\sqrt{14}} \text{ L.U}$$

The cartesian form of the perpendicular length from a point to a plane:

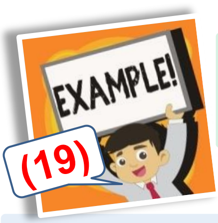
$$\because L = \frac{|\overrightarrow{BA} \cdot \vec{n}|}{\|\vec{n}\|} \quad L = \frac{|(x_1 - x_2, y_1 - y_2, z_1 - z_2).(a, b, c)|}{\sqrt{a^2+b^2+c^2}}$$

$$L = \frac{|(ax_1 + b\,y_1 + c\,z_1) - (ax_2 + b\,y_2 + c\,z_2)|}{\sqrt{a^2+b^2+c^2}}$$ ∵ the point B(x_2, y_2, z_2)

lies on the plane ax + by + cz + d = 0 $\quad \therefore d = -ax_2 - by_2 - cz_2$

$$\therefore L = \frac{|(ax_1 + b\,y_1 + c\,z_1 + d)|}{\sqrt{a^2+b^2+c^2}}$$

the cartesian form of the perpendicular length



Find the perpendicular length drawn from the point (1, 5, - 4) on the plane whose equation 3x - y +2z = 6

SOLUTION

$$\because L = \frac{|(ax_1 + b\,y_1 + c\,z_1 + d)|}{\sqrt{a^2+b^2+c^2}} = \frac{|(3(1) - 5 + 2(-4) - 6)|}{\sqrt{9+1+4}} = \frac{16}{\sqrt{14}}$$

EXAMPLE! (20)

Find the perpendicular length drawn from the point (-1, 4, 0) to the plane whose equation x - 2y – z = 4

SOLUTION

x - 2y – z - 4 = 0 $\because L = \frac{|(ax_1 + b\,y_1 + c\,z_1 + d)|}{\sqrt{a^2+b^2+c^2}}$

$= \frac{|(-1-2(4)-(0)-4)|}{\sqrt{1+4+1}} = \frac{13}{\sqrt{6}}$ L.u

The distance between two parallel planes :

EXAMPLE! (21)

Prove that the two planes x + 3y - 4z = 3, 2x + 6y - 8z = 4 are parallel and find the distance between them.

SOLUTION

$\overrightarrow{n_1} = (1, 3, -4)$, $\overrightarrow{n_2} = (2, 6, -8)$ $\frac{a_1}{a_2} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$

$\frac{c_1}{c_2} = -\frac{4}{-8} = \frac{1}{2}$ $\because \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

∴ the two planes are parallel to find the perpendicular distance in P(1):

let x = 0, y = 0, $z = -\frac{3}{4}$ the point $(0,0,-\frac{3}{4}) \in P(1)$

$L = \frac{\left|\left(2(4)+6(0)-8\left(-\frac{3}{4}\right)-4\right)\right|}{\sqrt{4+36+64}} = \frac{2}{\sqrt{104}} = \frac{\sqrt{26}}{26}$ L.u

EXAMPLE! (22)

Prove that the two planes : 3x + 6y + 6z = 4 , x + 2y + 2z = 1 are parallel and find the distance between them.

SOLUTION

$\overline{n_1} = (3, 6, 6)$, $\overline{n_2} = (1, 2, 2)$ $\frac{a_1}{a_2} = \frac{3}{1} = 3$ $\frac{b_1}{b_2} = \frac{6}{2} = 3$

$\frac{c_1}{c_2} = -\frac{6}{2} = 3$ $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ∴ P(1) // P(2)

in the P(1) : let x = 0, y = 0 , $z = \frac{4}{6} = \frac{2}{3}$

the point $(0,0,\frac{2}{3}) \in P(1)$ $L = \frac{\left|\left((0)+2(0)+2\left(\frac{2}{3}\right)-1\right)\right|}{\sqrt{1+4+4}} = \frac{\frac{1}{3}}{\sqrt{9}} = \frac{1}{9}$ i. u

The equation of the plane by using the intercepted parts from the coordinate (cartesian) axes.

* If the plane intersect the coordinate axes at the points $(x_1, 0, 0)$,$(0, y_1, 0)$ $(0,0,z_1)$ then the equation of the plane is in the form :

→ the equation of the plane in terms of the intercepted parts from axes. $\frac{x}{x_1} = \frac{y}{y_1} = \frac{z}{z_1} = 1$

EXAMPLE! (23)

Prove that the equation of the plane in terms of the intercepted parts from axes is $\frac{x}{x_1} = \frac{y}{y_1} = \frac{z}{z_1} = 1$

SOLUTION

$\overrightarrow{n_1} = (1 , 3 , -4)$, $\overrightarrow{n_2} = (2 , 6 , -8)$ $\frac{a_1}{a_2} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$

$\frac{c_1}{c_2} = -\frac{4}{-8} = \frac{1}{2}$ $\because \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

∴ the two planes are parallel to find the perpendicular distance in P(1):

let x = 0, y = 0,z = $-\frac{3}{4}$ the point $(0,0,-\frac{3}{4}) \in P(1)$

$$L = \frac{\left|\left(2\,(4) + 6\,(0) - 8\left(-\frac{3}{4}\right) - 4 \right)\right|}{\sqrt{4+36+64}} = \frac{2}{\sqrt{104}} = \frac{\sqrt{26}}{26} \text{ L.u}$$

EXAMPLE! (24)

Find the equation of the plane which intersects the coordinate axes x, y, z the parts 2 , -3 , 5 respectively.

SOLUTION

equation of the plane $\frac{x}{x_1} + \frac{y}{y_1} + \frac{z}{z_1} = 1 \rightarrow \frac{x}{2} - \frac{y}{3} + \frac{z}{5} = 1$

EXAMPLE! (25)

Find the parts intercepted by the plane 2x + 3y - z = 6 from the axes.

SOLUTION

$\because 2x + 3y - z = 6 \quad (\div 6) \rightarrow \frac{2x}{6} + \frac{3y}{6} - \frac{z}{6} = 1 \rightarrow \rightarrow \frac{x}{3} + \frac{y}{2} - \frac{z}{6} = 1$

the parts 3, 2, -6 respectively.

EXAMPLE! (26)

If the plane 3x + 2y + 4z = 12 cuts the axes x , y , z at the points A,B,C Find the area of triangle A B C.

SOLUTION

$3x + 2y + 4z = 12 \longrightarrow \rightarrow \frac{x}{4} + \frac{y}{6} + \frac{z}{3} = 1$ the parts (4 , 6 , 3)

$S = \frac{5 + 2\sqrt{15} + 3\sqrt{5}}{2}$

Let the sides of the triangle be m, n, o

$A = S\sqrt{S(S - m)(S - n)(S - o)} = 5\sqrt{11}$ unit area

Summary

Direction Vector:

1) If l,m and n are direction cosines of a straight line , So vector $\vec{d}$ = t (l, m, n) represents direction vector of the straight line and it's denoted by symbol $\vec{d}$ = (a, b, c) and numbers (a, b, c) called the direction ratios of straight line.

2) Direction vector of straight line take several equivalent forms as $\vec{d}$ = 2(l, m, n) = 3(l, m, n) = - 4(l, m, n) Equation of straight line:
Equation of straight line which passes through point (x_1, y_1 , z_1) and vector $\vec{d}$ = (a, b, c) is direction vector to it

Vector form $\vec{r}$ = (x_1, y_1 , z_1) + t (a, b, c) Parametric Form: x = x_1 + ta , y = y_1 + t c

Cartesian Form: $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$

Angle between two straight lines:

If d_1 & d_2 are direction vectors of two straight lines then the measure of the smallest angle between the two straight lines is $\cos\theta = \frac{|\overline{d}_1.\overline{d}_2|}{\|\overline{d}_1\|\ \|\overline{d}_2\|}$

and if (l_1, m_1, n_1) and (l_2, m_2, n_2) are direction cosines of two straight lines then: cos θ $= |l_1 l_2 + m_1 m_2 + n_1 n_2|$Parallel and perpendicular conditions of two straight lines: if $\overline{d}_1$ = (a_2, b_1,c_l) & $\overline{d}_2$ = (a_2, b_2, c_2) are direction vectors of two straight lines then: The two straight lines are parallel if: $\overline{d}_1$ = k$\overline{d}_2$ or $\overline{d}_1 \times \overline{d}_2 = \overline{o}$ or $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

The two straight lines are perpendicular if: a_1 a_2 + b_1 b_2 + c_1 c_2 = 0

Equation of a plane:

The equation of a plane passing through point (x_1,y_1,z_1) and vector $\vec{n}$ = (a, b, c) is perpendicular to the plane.

The vector Form: $\vec{n} \cdot \vec{r} = \vec{n}$. ($x_1$, y_1, z_1)

The standard Form: a(x - x_1) + b(y - y_1) + c(z – z_1) = 0

General Form: ax + by + cz + d = 0

Angle between two planes: If n_1 = a_1,b_1,c_1, & n_2 = a_2,b_2,C_2 are perpendicular vectors to two planes, then the measure of the angle between the two planes is given by the relation:

$$\cos\theta = \frac{|\overline{n}_1.\overline{n}_2|}{\|\overline{n}_1\|\ \|\overline{n}_2\|} \qquad \text{where } 0 \le \theta$$

The two parallel and perpendicular planes:

If $\overline{n}_1$ = (a_1,b_1,c_1) & $\overline{n}_2$ = (a_2, b_2,c_2)

are perpendicular vectors to two planes, then The conditions for the two planes to be parallel are:

$\overline{n}_1 \,//\, \overline{n}_2$ or $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

and to be perpendicular are: $\overline{n}_1 \cdot \overline{n}_2 = 0$ or $a_1a_2 + b_1 b_2 + c_1c_2 = 0$

The length of the perpendicular drawn from a point to a plane:

The length of the perpendicular drawn from point $A(x_1,y_1,z_1)$ to a plane passing through point B (x_2, y_2,z_2) and vector $\overline{n}_1 = (a, b, c)$ is perpendicular to plane.

The vector Form: L $|\overline{BA} \cdot \overline{n}|$

The cartesian Form: $L = \frac{|ax_1 + b\,y_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}}$

Answer the following question:

S.B.2017

1) find the different forms of the equation of the plane passing through the point (2, - 1, 0) and the vector is $\vec{n} = 4\,\hat{i} + 10\,\hat{j} - 7\,\hat{k}$ is normal to it

[4 x + 10 y – 7 z + 2 = 0]

S.B.2017

2) find the general equation of the plane passing through the origin point and the vector $\vec{n} = \hat{i} + 2\,\hat{j} - 3\,\hat{k}$ is normal to it

[x + 2 y – 3 z = 0]

S.B.2017

3) find the different forms of the equation of the plane passing through the three point A (2, - 1, 0), B (- 1, 3, 4) and C (3, 0, 2)

[(4, 10, - 7) • $\vec{r}$ = - 2)]

S.B.2017

4) find the equation of the plane which passes through the point (2, 1, 4) and is :

(a) parallel to the plane 2 x + 3y + 5 z = 1 [2 x + 3 y + 5 z – 27 = 0]

(b) perpendicular to the straight line passing through the two points (3, 2, 5) and (1, 6, 4) [2 x - 4 y + z – 4 = 0]

(c) perpendicular to each of the planes 7x + y + 2 z = 6 and 3 x + 5 y – 6 z = 8 [- x + 3 y + 2 z – 9 = 0]

S.B.2017

5) if the plane X contains the points A (1, 4, 2), B (1, 0, 5) and C (0, 8, -1) and the plane Y contains the point D (2, 2, 3) and the vector $\vec{n} = \hat{i} + 2\,\hat{j} - 2\,\hat{k}$ is perpendicular to it

(a) find the cartesian equation of the X- plane. [3y + 4z – 20 = 0]

(b) find the cartesian equation of the Y- plane [x + 2y + 2z – 12 = 0]

(c) what are the values of t and f if the point (t, 0, f) belongs to each of the two planes X, Y ? [f = 5 , t = 2]

(d) find the vector equation of the line of intersection of the two planes X, Y [$\vec{r}$ = (2, 0, 5) + t (- 2, 4, - 3)]

(e) if the point (1, 1, p) is equidistant from the two planes X, Y, find the possible values of p $\left[\frac{48}{11} \text{ or } 3\right]$

S.B.2017

6) find the different forms of the equation of the plane that intercepts 2, 4, 5 from the coordinate axes x, y , z respectively

[(10, 5, 4) • $\vec{r}$ = 20]

S.B.2017

7) find the equation of the plane which contains the straight line L_1 and is parallel to the straight line L_2 where

$L_1 : \vec{r} = (0, 3, -5) + t_1 (6, -2, -1)$

$L_2 : \vec{r} = (1, 7, -4) + t_2 (1, -3, 3)$ [$9x + 19y + 16z + 23 = 0$]

S.B.2017

8) if a plane intersects the coordinate axes at the point A, B, C and the point (p, q , r) is the point of intersection of the medians of triangle ABC, prove that the equation of the plane is $\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 3$

S.B.2017

9) if the point A, B, C and D are in space, where their position vectors with respect to the origin point are $-\hat{j} + \hat{k}$, $2\hat{i} - \hat{j} + 3\hat{k}$, $-\hat{i} - 2\hat{j} + 2\hat{k}$ and $7\hat{i} - 4\hat{j} + 2\hat{k}$ respectively

(a) find the normal vector to the plane ABC [$2\hat{i} - 4\hat{j} - 2\hat{k}$]

(b) show that the length of the perpendicular from D to the plane ABC equals $2\sqrt{6}$

(c) show that the two planes ABC and DBC are orthogonal .

(d) find the equation of the line of intersection of the two planes ABC and ODB [$\vec{r} = (2, -1, 3) + t (38, -18, 74)$]

S.B.2017

10) find three point in space belonging to each of the following planes:

(a) $x = 3$ [(3, 0, 0), (3, 1, 2)and (3, - 1 , 4)]

(b) $y = -2$ [(0, - 2, 0), (1, - 2, 3)and (2, - 2 , 4)]

(c) $x + 3y = 5$ [(2, 1, 0), (- 1, 2, 0)and (8, - 1 , 0)]

(d) $2x - y + 3z = 4$ [(1, 1, 1), (0, 0, $\frac{4}{3}$)and (0, - 1 , 1)]

S.B.2017

11) prove that the point A (2, 3, 1) and the straight line $L : \vec{r} = (3\hat{i} + \hat{j} + 3\hat{k}) + t\ (\hat{i} - 2\hat{j} + 2\hat{k})$ lie on the plane whose equation is $\vec{r} \bullet (2\hat{i} - \hat{k}) = 3$

S.B.2017

12) find the distance between the point (2, 1, - 1) and the plane

$\vec{r} \bullet (\hat{i} - 2\hat{j} + 4\hat{k}) = 9$ $\left[\frac{13\sqrt{21}}{21}\right]$

S.B.2017

13) prove that the two planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ are parallel, then find the distance between them $\left[\frac{21}{6}\right]$

MOE 2017

14) find the projection of the point (1, 2, 3) on the plane

$x + 2y + 4z = 59$ [(3, 6, 11)]

S.B. 2017

15) find the point of intersection of the planes $2x + y - z = -1$, $x + y + z - 2 = 0$ and $3x - y - z = 6$ $\left[\left(2, \frac{-5}{2}, \frac{5}{2}\right)\right]$

S.B. 2017

16) if the length of the perpendicular drawn from the point A (0, - 1 , 2) to the plane $\sqrt{2}\,x + y = z + k = 0$ equals 2 units of length , find the value of k . [7 or – 1]

S.B. 2017

17) if the plane $2x - y - 2z + 12 = 0$ intersects with the sphere $(x+3)^2 + (y+2)^2 + (z-1)^2 = 15$, find the area of the cross section (trace) [11 π squared units]

S.B. 2017

18) if the plane $2ax - 3ay + 4az + 6 = 0$ passes through the mid-point of the line segment joining the centers of the two spheres $x^2 + y^2 + z^2 - 6x + 8y - 2z = 13$ and $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$, find the value of a [- 2]

S.B. 2017

19) find the measure of the angle between each of the following pairs of planes :

(a) $p_1 : 2x - y + z = 5$, $p_2 : 3x + 2y - 2z = 1$ [78° 34' 42"]

(b) $p_1 : \vec{r} \bullet (2, 1, -1) = 4$, $p_2 : \vec{r} \bullet (3, -2, 0) = 7$ [63° 4' 10"]

(c) $p_1 : y = 4$, $p_2 : x - 3y + 5z = 1$ [59° 31' 47"]

(d) $p_1 : 3x - y = 5$, $p_2 : x - 2y = 4$ [45°]

(e) $p_1 : 2x + 2y + 7z = 8$, $p_2 : 3x - 4y + 4z = 5$ [57° 28']

S.B. 2017

20) find the cartesian equation of the plane whose equation is $(x, y, z) = (2, 3, 5) + t_1(-1, 3, 4) + t_2(6, 4, -2)$, where t_1 and t_2 are parameters [$10x - 22y + 19z - 49$]

MOE 2017

21) a sphere of center M (2, - 1, -2) and radius length units is placed on the plane $2x + 6y - 3z + k = 0$, find the value of k [17, or – 25]

MOE 2017

22) find the equation of the line of intersection of the two planes $x + 2y - 2z = 1$ and $2x + y - 3z = 5$ [$\vec{r} = (3, -1, 0) + t(-4, -1, -3)$]

MOE 2017

23) prove that the two planes $2x + y + 2z = 8$ and $4x + 2y + 4z = 10$ are parallel , then find the distance between them . [1 units of length]

S.B.2017

24) prove that the straight line $\vec{r} = \hat{k} + t(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$ is perpendicular to the plane $x + \frac{3}{2}y + 2z = 5$

S.B. 2017

25) find the coordinates of the point of intersection of the straight line $\vec{r} = \hat{k} + t\,(2\,\hat{\imath} + \hat{\jmath} + \hat{k})$ with the plane $\vec{r} \bullet \hat{\imath} = 4$ [(4, 2, 3)]

S.B. 2017

26) find the coordinates of the point of intersection of the straight line passing through the two points (3, - 4 , -5) and (2, -3, 1) with the plane passing through the points (2, 2, 1), (3, 0, 1) and (4, -1 0) [(1, - 2, 7)]

S.B. 2017

27) find the coordinates of the point of intersection of the straight line $\vec{r}$ = (2, - 1, 2) + t (3, 4, 2) with the plane $\vec{r} \bullet$ (1, - 1, 1) =5

[(2, -1, 2)]

S.B. 2017

28) prove that the straight line $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z}{3}$ intersects the plane 3x + 2y + z – 8 = 0 at a point and find the measure of the inclination angle of the line to the plane [30°]

S.B. 2017

29) find the point of intersection of the straight line x = y = z and the plane x + 2 y + 3 z = 12 [(2, 2, 2)]

MOE 2017

30) find the measure of the angle included between the straight line L: $\frac{x-3}{\sqrt{2}} = \frac{y-1}{1} = \frac{-z-2}{1}$ and the plane $\sqrt{2}$ x – y – z + 5 = 0 [30°]

MOE 2017

31) if the straight line x = 2 + t, y = - 1 + 2 t , z = - 3 t is parallel to the plane 11 x – 4 y + z = 0, find the distance between them

$\left[\frac{13\sqrt{138}}{69}\ \textit{unit of length}\right]$

32) consider the points A (2, 1, 0), B (1, 2, 2), C (3, 3, 1) and D (1, 1, 4):

i) verify the points A , B , C determine a plane and x – y + z – 1 = 0 is a cartesian equation for it .

ii) Show that △ ABC is equilateral and prove that its area is $\frac{3\sqrt{3}}{2}$ units of area.

iii) find the parametric form of the equations for the straight line that passes through D , perpendicular to the plane ABC .

[x = 1+ t, y = 1 – t, z = 4 + t]

iv) the point E is the projection of D on the plane ABC :

(a) determine the coordinates of the point E , then calculate the distance between D and the plane ABC [(0, 2, 3) , $\sqrt{3}$ units length]

(b) determine the centers of the two spheres that touch plane ABC at E and the radius of each of them is $\sqrt{3}$ units of length [(1, 1, 4), (-1 , 3 ,2)]

c) calculate the volume of the pyramid ABCD $\left[\frac{3}{2}\ \textit{cubic unites}\right]$

33) consider the points A (2, 4, 1), B (0, 4, - 3), C (3, 1, - 3) and D (1, 0, -2):

Answer each of the following with (true) or (false) giving reasons:

i) the point A, B, C are not collinear ( )

ii) 2x + 2 y - z – 11 = 0 is a cartesian equation for the plane ABC ( )

iii) the point E (3, 2, - 1) is the projection of the point D on plane ABC ( )

iv) the two straight line $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ are coplanar ( )

v) x = t – 3, y = - t , z = t + 1 are the parametric equations of $\overleftrightarrow{CD}$ where t ∈ $\Re$ ( )

34) consider the points A (1, - 1, - 2), B (1, - 2, - 3) and C (2, 0, 0)

i) (a) prove that the points A, B, C are not collinear.

(b) write the vector equation for the plane ABC

(c) verify that x + y – z – 2 = 0 is a cartesian equation for the plane ABC

ii) consider the two plane P and Q defined by the equations P : x - y – 2 z + 5 = 0 and Q : 3 x + 2 y –z + 10 = 0 prove that the two planes P and Q intersect at the straight line L whose parametric equations are x = t – 3, y = - t , z = t + 1 where t ∈ $\Re$

iii) determine the intersection of the planes ABC, P, Q [(-9 , 6 , -5)]

35) consider the point A (- 1 , 1, 3), B (1, 0, - 1), C (2, -1, 1) and D (2, 0, -1) and the plane P whose equation is 2 y +z + 1 = 0 Let L be a straight line whose parametric equation are x = - 1 , y = 2 + t , z = 1 – 2 t where t ∈ $\Re$

i) write the parametric form of the equations for $\overleftrightarrow{BC}$ and verify that $\overleftrightarrow{BC}$ lies in the plane P. [x = 1+t, y = – t , z = - 1 + 2t]

ii) show that the two straight lines L, $\overleftrightarrow{BC}$ are not coplanar (skew)

iii) calculator the distance between point A and the plane P

$\left[\frac{6\sqrt{5}}{5}\ unit\ of\ length\right]$

(b) show that D is a point in P and △ BCD is a right-angled triangle

iv) show that ABCD is a pyramid and calculate its volume

[1 cubic unites]

36) Consider the points A (2, 1, -1), B (1, - 1, 3), C $\left(-\frac{3}{2}, -2, 1\right)$ and D $\left(\frac{7}{2}, -3, 0\right)$ and let I be the mid-point of $\overline{AB}$

i) (a) Find the coordinates of I. $\left(\frac{3}{2}, 0, 1\right)$

(b) Prove that the plane P: 2 x + 4y - 8z + 5 = 0 is perpendicular bisector of $\overline{AB}$.

ii) Write the parametric form of the equations of the straight-line L that passes through the point C and $\vec{u}$ = (1, 2, - 4) is a direction vector for it.

[x = - $\frac{3}{2}$ + t, y = –2 + 2 t, z = 1 - 4t]

iii) (a) Find the coordinates of E which is the point of intersection of plane P and the line L. $\left[\left(-\frac{7}{6}, -\frac{4}{3}, -\frac{1}{3}\right)\right]$

(b) Show that L, $\overleftrightarrow{AB}$ are coplanar and deduce that △ IEC is a right triangle

iv) (a) Show that $\overleftrightarrow{ID}$ is perpendicular to each of $\overleftrightarrow{AB}$, $\overleftrightarrow{IE}$.

(b) Calculate the volume of the pyramid DIEC. $\left[\frac{28}{9}\text{ cubic unites}\right]$

37) If the points A (1,1,0), B (2, 1, 1), C (-1, 2, - 1), answer the following:

i) (a) Prove that A, B, C are not collinear.

(b) Show that the equation of the plane ABC is x + y – z - 2 = 0.

ii) If the equations of the planes X and Y are x + 2 y - 3z + 1 = 0 and 2 x + y - z - 1 = 0 respectively and the straight line L passes through the point C (0, 4,3) and $\vec{u}$ = (-1, 5, 3) is a direction vector for it.

(a) Write the parametric form of the equations of straight-line L.

[x = - t, y = 4 +5 t, z = 3 +3t]

(b) Prove that the line of intersection of the two planes X and Y is the straight line L.

iii) Determine the intersection of the three planes ABC, X, Y.

[(-1,9,6)]

38) Given that the equation of the plane X: x- 2 y + z + 3 = 0, answer the following:

i) Determine the point A which is the point of intersection between plane X and x-axis. [(-3,0,0)]

ii) If B (0,0,-3) and C (-1,- 4, 2):

(a) Prove that the point B lies on plane X.

(b) Calculate AB. [3 $\sqrt{2}$ unites of length]

(c) Calculate the distance between the point C and the plane X

[2 $\sqrt{6}$ unites of length]

iii) (a) Write the parametric form of the equation of the straight line passes through the point C and perpendicular to plane X.

[x = - 1 + t, y = – 4 - 2 t, z = 2 + t]

(b) Prove that point 4 belongs to a straight-line L.

(c) Calculate the area of A ABC. [$6\sqrt{3}$ squared unites]

39) Prove that $\left(\vec{A} \bullet \vec{B}\right) + \left\|\vec{A} \times \vec{B}\right\| \le \sqrt{2}\ \left\|\vec{A}\right\|\ \left\|\vec{B}\right\|$