S.B.2017

27) ABCD is square with side length 12 cm $\vec{C}$ is a unit vector perpendicular to its plane , find:

(a) $\overrightarrow{AB} \bullet \overrightarrow{AC}$ [144]

(b) $\overrightarrow{AB} \times \overrightarrow{CA}$ [- 144 $\vec{C}$]

(C) $\overrightarrow{BC} \bullet \overrightarrow{AD}$ [144]

(d) $\overrightarrow{BD} \times \overrightarrow{AC}$ [- 288 $\vec{C}$]

(E) $\overrightarrow{AB} \bullet \overrightarrow{BC}$ [0]

(F) $\overrightarrow{AB} \times \overrightarrow{BC}$ [144 $\vec{C}$]

S.B.2017

28) find the unit vector perpendicular to the plane which contains the two vectors $\vec{A} = 2\,\hat{\imath} - 3\,\hat{\jmath} + \hat{k}$ and $\vec{B} = -\,\hat{\imath} + 2\,\hat{\jmath} - 3\,\hat{k}$

$\left[\left(\frac{7\sqrt{3}}{15}\hat{\imath} + \frac{5\sqrt{3}}{15}\hat{\jmath} + \frac{\sqrt{3}}{15}\hat{k}\right)\right]$

S.B.2017

29) calculate the area of triangle DEF in each of the following:

(a) L (5, 1, -2) , E (4, - 4 , 3) F (2, 4, 0) [16.72 square units]

(b) D (4, 0, 2) , E (2, 1 , 5) F (- 1, 0, - 1) [≡ 10.9 square units]

S.B.2017

30) calculate the area of the parallelogram LMNE in each of the following:

(a) L (1, 1,), M (2, 3) N (5, 4,) [5 square units]

(b) L (2, 1, 3), M (1, 4 , 5) N (2, 5, 3) [$4\sqrt{5}$ square units]

S.B.2017

30) find the volume of the parallelepiped in which $\vec{A}$, $\vec{B}$, $\vec{C}$ are three adjacent edges $\vec{A} = (1, 1, 3)$, $\vec{B} = (2, 1, 4)$ $\vec{C} = (5, 1, -2)$ [9 cubic units]

S.B.2017

31) in each of the following, show whether the two given vectors are parallel, perpendicular or otherwise

(a) $\vec{A} = (0, 2, 2)$, $\vec{B} = (3, 0, -4)$ [neither perpendicular nor parallel]

(b) $\vec{A} = 10\,\hat{\imath} + 40\,\hat{\jmath}$, $\vec{B} = -3\,\hat{\imath} + 8\,\hat{k}$

[neither perpendicular nor parallel]

(c) $\vec{A} = -2\,\hat{\imath} + \hat{\jmath} - 2\,\hat{k}$, $\vec{B} = 8\,\hat{\imath} - 4\,\hat{\jmath} + 8\,\hat{k}$

[not perpendicular but parallel]

S.B.2017

32) if A = (0, 0, 1), B = (1, 0, 0) C = (0, 1, 0) , find the orthogonal unit vector to the plane ABC $\left[\frac{\sqrt{3}}{3}\hat{\imath} + \frac{\sqrt{3}}{3}\hat{\jmath} + \frac{\sqrt{3}}{3}\hat{k}\right]$

S.B.2017

33) prove each of the following where $\vec{A}$, $\vec{B} \in \Re^3$

(a) $\|\vec{A} \times \vec{B}\|^2 + (\vec{A} \bullet \vec{B})^2 = \|\vec{A}\|^2 \ \|\vec{B}\|^2$

(b) if $\vec{A} \bullet \vec{B} = 0$ and $\vec{A} \times \vec{B} = \vec{0}$, then $\vec{A} = \vec{0}$ or $\vec{B} = \vec{0}$

Choose the correct answer in each of the following:

S.B.2017

34) $i\hat{\imath} \times \hat{\jmath} =$ ……..

a) $\vec{0}$ (b) 0 (c) 1 (d) $\hat{k}$

S.B.2017

35) if the vectors (2, k, - 3) and (4, 6, - 6) are parallel, then k = …..

a) 6 (b) 3 (c) - 3 (d) 1

S.B.2017

36) if $\vec{A} \mathbin{//} \overrightarrow{B}$, then $\|\overrightarrow{A} \times \vec{B}\| =$ ……

a) 0 (b) 1 (c) $\|\overrightarrow{A}\|$ (d) $\|\vec{B}\|$

S.B.2017

37) if $\vec{A}$, $\vec{B}$,$\vec{C}$ are non-zero vectors , $\vec{A} \times \vec{C} = \vec{0}$ and $\overrightarrow{A} \bullet \vec{B} = 0$, then $\overrightarrow{B} \bullet \vec{C} =$ …….

a) 0 (b) 1 (c) $\overrightarrow{A}$ (d) $\|\vec{B}\|$

Complete the following:

S.B.2017

38) if $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ form a right-hand system of unit vectors then $\hat{\jmath} \times \hat{k} =$ …

S.B.2017

39) if $\vec{A}$, $\vec{B}$ are non-zero vectors and $\vec{A} \times \vec{B} = \vec{0}$, then $\vec{A}$ and $\vec{B}$ are …..

S.B.2017

40) if $\vec{A} = (k, 3, -4)$, $\vec{B} = (-2, 9, m)$ and $\vec{A} \mathbin{//} \vec{B}$, then k = ….. , m = …..

S.B.2017

41) if $\overrightarrow{A} \bullet \vec{B} = \|\overrightarrow{A} \times \vec{B}\|$, then the measure of the angles between the two vectors $\vec{A}$ and $\vec{B}$ equal

Answer the following:

S.B.2017

42) find the volume of the parallelepiped in which three adjacent sides are represented by the vectors $\vec{A} = (1, -1, 2)$, $\vec{B} = (3, -2, 0)$ and $\vec{C} = (0, 2, 4)$

S.B.2017

43) $\vec{A}$, $\vec{B}$,$\vec{C}$ are three mutually perpendicular unit vectors, if $\vec{A} = \left(\frac{4}{5}, 0, \frac{3}{5}\right)$ and $\vec{B} = \left(\frac{3}{5}, 0, \frac{-4}{5}\right)$ find $\vec{C}$

S.B.2017

44) find the volume of the parallelepiped in which three of its adjacent sides are represented by the vectors $-12\,\hat{\imath} - 3\,\hat{k}$, $3\,\hat{\jmath} - \hat{k}$ and $2\,\hat{\imath} + \hat{\jmath} - 15\,\hat{k}$

S.B.2017

45) find the area of the triangle whose vertices are the points A (4, 2, - 3), B (7, - 2, - 2) and C (1, 8, - 3), then find a unit vector perpendicular to the plane of the triangle

46) prove that $(\vec{A} + \vec{B})(\vec{A} - \vec{B}) = -2\vec{A} \times \vec{B}$

47) find the all vectors that are perpendicular to each $\vec{a}$ and $\vec{b}$, where $\vec{A} = (2, 3, -1)$ and $\vec{B} = (1, -2, 2)$, hence; find the vector whose normal is 5 units of length and perpendicular to each $\vec{a}$ and $\vec{b}$

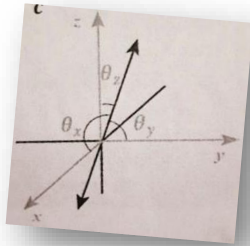
48) find the volume of the triangular pyramid ABCD where A (0, 0, 1), B (2, 3, 0) and C (-1, 2, 1), D (1, -2, 4)

49) if A (-1, 2, 1), B (0, 1, 4) and C (k, -1, -2) are the vertices of a triangle whose surface area equals $\sqrt{118}$ squared units, find the value of k

Direction vector of straight line in space:

If θ_x , θ_y , θ_z are direction angles of straight line in space , let $\overrightarrow{u}$ be unit vector in direction of this line $\overrightarrow{u}$ = ($\cos\theta_x$,$\cos\theta_y$,$\cos\theta_z$)

let $\overrightarrow{u}$ = (l, m, n), let the vector $\overrightarrow{d}$ be parallel to the vector $\overrightarrow{c}$



$\therefore \overrightarrow{d} = k\,\overrightarrow{u}$, $\overrightarrow{d}$ = (l, m, n)

$\overrightarrow{d}$ = k (l, m, n) , $\overrightarrow{d}$ = (a, b, c)

Where:

(1) a, b, c are proportional to l, m, n and $k \in R^+$

(2) a, b, c are called direction ratios (direction numbers)

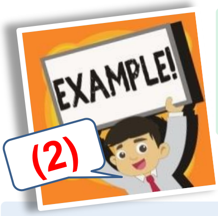
if $\overrightarrow{c} = \left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$ are direction cosines of straight lines. find number of direction vectors that are parallel to vector $\overrightarrow{c}$

SOLUTION

Let the direction vector be $\overrightarrow{d}$, $\overrightarrow{d} = k\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$

$\because k \in R^*$ or $k \in R \sim |0|$

$\therefore$ the straight line has in finite number of parallel direction vectors.

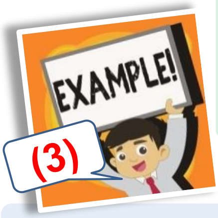


find the direction vector of the straight line passing by the two points A(- 2, 3, 1), B (0, 4, - 2)

SOLUTION

The direction vector of the straight line = $\overrightarrow{AB} = \overrightarrow{B} - \overrightarrow{A}$

= (0, 4, - 2) – (- 2, 3, 1) $\therefore \overrightarrow{u}$ = (2, 1, - 3)



Find the direction vectors in each of the following straight lines

(a) the straight line passing though the origin point and the point (- 1, 2, - 2)

(b) the straight line passing by the two points c (0, - 2, 3), d (1, 1, - 1)

SOLUTION

(a) the direction vector $\overrightarrow{u} = \overrightarrow{OA}$, $\overrightarrow{u} = \overrightarrow{A} - \overrightarrow{O} \rightarrow \overrightarrow{u} = \overrightarrow{A} \rightarrow \overrightarrow{u}$ = (- 1, 2, - 2)

(b) the direction vector $\overrightarrow{u} = \overrightarrow{CD}$, $\overrightarrow{u} = \overrightarrow{D} - \overrightarrow{C}$

$\rightarrow \overrightarrow{u}$ = (1, 1, - 1) - (0, - 2, 3) $\rightarrow \overrightarrow{u}$ = (1, 3, - 4)

What can you say about the straight line whose vector direction $\overrightarrow{u}$ = (a, b, c) ?

EXAMPLE! (4) find the vector directions of each of the coordinate axes

SOLUTION

x - axis ⟶ (a, 0, 0) , y - axis ⟶ (0, b, 0) , z - axis ⟶ (0, 0, c)
where a, b, c ∈ R^*

**** vector form of the equation of a straight line in space***

If L is a straight line in the space its direction vector is $\vec{d}$ = (a, b, c)
and passing by the point A whose position vector $\vec{A}$ = (x_1, y_1, z_1)
if the point B is any point in the straight line whose position vector
$\vec{r}$ = (x, y, z) then form the opposite figure:

$\vec{r} = \vec{A} + \overrightarrow{AB}$ ∴ $\overrightarrow{AB} \parallel \vec{d}$ ⟶ $\overrightarrow{AB} = t\,\vec{d}$

$\vec{r} = \vec{A} + t\,\vec{d}$ ⟶ (the vector form of the equation of straight line)

EXAMPLE! (5) find the vector form of a straight line passing by the point (3, - 1, 0) and the vector (- 2, 4, 3) is a direction vector of it

SOLUTION

∵ (3, - 1 , 0) is a point on the straight line ∴ $\vec{A}$ = (3, - 1 , 0)
∵ (- 2, 4, 3) is a direction vector of the straight line ∴ $\vec{d}$ = (- 2, 4, 3)
$\vec{r} = \vec{A} + \overrightarrow{t\,d}$ ⟶ $\vec{r}$ = (3, - 1, 0) + t (- 2, 4, 3)

Remark

T is a real number not constant at unique value but it has many different real values and called parameter in this case and at each parameter t it possible to find a point on the straight line .

For example

In the previous example when t = 1 then $\vec{r}$ = (3, - 1, 0) + (- 2, 4, 3)
$\vec{r}$ = (1, 3, 3) represent a position vector of another point on a straight line at
t = 2 ⟶ $\vec{r}$ = (3, - 1, 0) + (- 4, 8, 6) ⟶ $\vec{r}$ = (- 1, 7, 6)
represent a position vector of another point on a straight line.

EXAMPLE! (6)

find the vector form of equation of straight line passing by the points (4, - 2, 5) and the vector (1, - 2, 2) is a direction vector of it

SOLUTION

The point (4, - 2, 5) represent a point on straight line

$\therefore \overrightarrow{A}$ (4, - 2, 5) the vector (1, - 2, 2) is a direction vector of a straight line

$\therefore \overrightarrow{d}$ = (1, - 2 , 2) $\therefore$ the vector form of the equation of straight line $\overrightarrow{r} = \overrightarrow{A} + \overline{t\,d}$ = (4, -2, 5) + t(1, -2, 2)

The parametric equation of a straight line in space:

From the vector equation of a straight line: $\overrightarrow{r} = \overrightarrow{A} + \overline{t\,d}$

By putting $\overrightarrow{r}$ = (x, y, z), $\overrightarrow{A}$ = (x_1, y_1, z_1), $\overrightarrow{d}$ = (a, b, c) then

(x, y, z) = (x_1, y_1, z_1), + (a, b, c)

$x = x_1 + at$, $y = y_1 + b\,t$, $z = z_1 + c\,t$ the parametric equation of st. line

EXAMPLE! (7)

find the parametric equation of the straight line passing through the point (2, - 1, 3) and the direction vector of this line is (- 2, 3, 1)

SOLUTION

$\overrightarrow{r} = \overrightarrow{A} + \overline{t\,d}$ $\rightarrow \overrightarrow{r}$ = (2, - 1, 3) + t (4, - 2, 5)

(x, y, z) = (2, - 1, 3) + t (4, - 2, 5)

$\therefore$ x = 2 + 4 t , y = - 1 – 2 t , z = 3 + 5 t

EXAMPLE! (8)

find the parametric equation of the straight line passing the origin point and the vector (- 2, 3, 1) is a direction vector of this line

SOLUTION

$\overrightarrow{r} = \overrightarrow{A} + \overline{t\,d}$, (x, y, z) = (0, 0, 0) + t (- 2, 3, 1)

x = - 2 t , y = 3 t , z = t

Cartesian equation of a straight line in space:

From a parametric equation of a straight line

$x = x_1 + t\,a$, $y = y_1 + t\,b$, $z = z_1 + t\,c$

$ta = x - x_1$ $ta = y - y_1$,$tc = z - z_1$

$t = \frac{x - x_1}{a}$, $t = \frac{y - y_1}{b}$,$t = \frac{z - z_1}{c}$

$\therefore \frac{x - x_1}{a}$, $\frac{y - y_1}{b}$, $\frac{z - z_1}{c}$ is the cartesian form of equation of straight line .

Remark

(1) when a = 0 then the cartesian form of equation of straight line is

$x = x_1$, $\frac{y-y_1}{b} = \frac{z-z_1}{c}$

(2) the equation of the line in the plane : ax + by + c = 0

(3) the equation : ax + by + cz + d = 0 represents the equation of the plane in the space.

(4) the direction ration are proportional to the direction cosines l,m,n then the Cartesian form of the equation of straight line on the form:

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$$

EXAMPLE! (9)

find the different form of equation of straight line passing by two point (2, - 1, 5), (- 3, 1, 4)

SOLUTION

Let A = (2, - 1, 5), B = (- 3, 1, 4)

$\vec{d} = \overrightarrow{AB} \rightarrow \vec{d} = \vec{B} - \vec{A} \rightarrow \vec{d}$ = (- 3, 1, 4) - (2, - 1, 5)

$\rightarrow \vec{d}$ = (- 5, 2, - 1)

The vector equation of a straight line $\vec{r} = \vec{A} + \overrightarrow{t\,d}$

$\vec{r}$ = (2, - 1, 5) + t (- 5, 2, -1)

The parametric equation of straight line

x = 2 – 5 t , y = -1 + 2t , z = 5 - t

the cartesian equation of straight line $\frac{x-2}{-5} = \frac{y+1}{2} = \frac{z-5}{-1}$

EXAMPLE! (10)

find the different form of equation of straight line passing the two point (3, 2, 0), (- 1, 3, 4)

SOLUTION

The vector form of equation of a straight line :

$\vec{r} = \vec{A} + \overrightarrow{t\,d}$ $\qquad \vec{d} = \overrightarrow{AB}$

$\vec{r}$ = (3, 2, 0) + t (- 4, 1, 4), $\vec{d}$ = (- 4, 1, 4) , $\vec{d} = \vec{B} - \vec{A}$

The parametric form of equation of straight line:

x = 3 – 4 t , y = 2 + t , z = 4 t

the cartesian equation of a straight line : $\frac{x-3}{-4}$ = y – z = $\frac{z}{4}$

EXAMPLE! (11)

find the different forms of equation of straight line $\frac{3x+1}{2} = \frac{y-1}{2} = \frac{5-z}{3}$

SOLUTION

Let $\frac{3x+1}{2} = \frac{y-1}{2} = \frac{5-z}{3} = t$

$\because \frac{3x+1}{2} = t \rightarrow x = -\frac{1}{3} + \frac{2}{3}t$, $\because \frac{y-1}{2} = t \rightarrow y = 1 + 2t$

$\frac{5-z}{3} = t \rightarrow z = 5 - 3t$ $\therefore (x, y, z) = \left(-\frac{1}{3}, 1, 5\right) + t\left(\frac{2}{3}, 2, -3\right)$

$\vec{r} = \left(-\frac{1}{3}, 1, 5\right) + t\left(\frac{2}{3}, 2, -3\right)$ the vector equation of straight line

Remark

The direction ratio are $\left(\frac{2}{3}, 2, -3\right)$ or (2, 6, -9) or ……

EXAMPLE! (12)

find the different forms of the straight line $\frac{x+4}{3} = \frac{2y+5}{2} = \frac{4-z}{4}$ then find a point lying of the line.

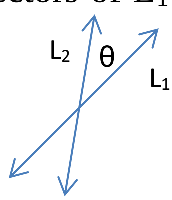
SOLUTION

Let $\frac{x+4}{3} = t \rightarrow x = -4 + 3t$ parametric equation of straight line

$\frac{2y+5}{2} = t \rightarrow y = -\frac{5}{2} + \frac{2t}{2} \rightarrow y = -\frac{5}{2} + t$

The angel between two straight lines in space:

* if L_1 , L_2 are two straight lines in the space the direction vectors of L_1 , L_2 are $\overrightarrow{d_1}$, $\overrightarrow{d_2}$



$\overrightarrow{d_1} = (a_1, b_1, c_1)$

$\overrightarrow{d_2} = (a_2, b_2, c_2)$ $\cos\theta = \frac{|\overrightarrow{d_1} \cdot \overrightarrow{d_2}|}{\|\overrightarrow{d_1}\| \, \|\overrightarrow{d_2}\|}$ $0 \le \theta \le \frac{\pi}{2}$

* if (l_1, m_1, n_1) , (l_2, m_2, n_2) are the direction cosines of the two straight lines then :

$\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$

find the measure of the angle between the two straight lines

$\overrightarrow{r_1} = (2, -1, 3) + t_1(-2, 0, 2)$ and $x = 1, \ \dfrac{y-4}{3} + \dfrac{z+5}{-3}$

SOLUTION

From the vector equation of 1st line : $\overrightarrow{d_1} = (-2, 0, 2)$

From the parametric equation of 2nd line : $\overrightarrow{d_2} = (0, 3, -3)$

$\therefore \cos\theta = \dfrac{|\overrightarrow{d_1} \cdot \overrightarrow{d_2}|}{\|\overrightarrow{d_1}\| \, \|\overrightarrow{d_2}\|} \quad = -6$

$\overrightarrow{d_1} \cdot \overrightarrow{d_2} = (-2, 0, 2) \cdot (0, 3, -3) = 0 + 0 - 6$

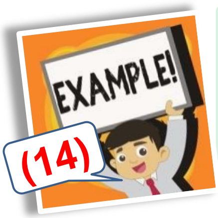
$\cos\theta = \dfrac{|-6|}{\sqrt{8}\,\sqrt{18}} = \dfrac{6}{2\sqrt{2} \times 3\sqrt{2}}$

$\|\overrightarrow{d_1}\| = \sqrt{4+0+4} = \sqrt{8}$

$\cos\theta = \dfrac{6}{12} = \dfrac{1}{2}$

$\|\overrightarrow{d_2}\| = \sqrt{9+0+9} = \sqrt{18}$

$\therefore \theta = 60°$



find the measure of the angle between the two straight lines :

$L_1 : \ x = 2 - 5t \ , \ y = 1 - t \ , z = 3 + 4t$

$L_2 = 1, \ \dfrac{x+1}{3} = \dfrac{2-y}{4} = \dfrac{z}{2}$

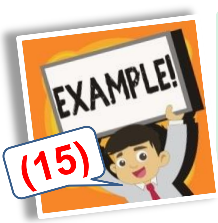
SOLUTION

From L_1 : $\overrightarrow{d_1} = (-5, -1, 4)$ From L_2 : $\overrightarrow{d_2} = (3, 4, 2)$

$\overrightarrow{d_1} \cdot \overrightarrow{d_2} = (-5, -1, 4) \cdot (3, 4, 2) = -15 - 4 + 8 = -11$

$\|\overrightarrow{d_1}\| = \sqrt{25+1+16} = \sqrt{42}$ $\quad \|\overrightarrow{d_2}\| = \sqrt{9+16+4} = \sqrt{29}$

$\cos\theta = \dfrac{|\overrightarrow{d_1} \cdot \overrightarrow{d_2}|}{\|\overrightarrow{d_1}\| \, \|\overrightarrow{d_2}\|}$, $\cos\theta = \dfrac{|-11|}{\sqrt{42}\,\sqrt{29}}$ $\quad \therefore \theta = 71° \, 37' \, 40''$



find the measure of the angle between the two lines whose direction cosines are :

$\left(\dfrac{5}{13\sqrt{2}}, \dfrac{-12}{13\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right) , \left(\dfrac{-3}{5\sqrt{2}}, \dfrac{4}{5\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$

SOLUTION

$\cos\theta = |L_1 L_2 + m_1 m_2 + n_1 n_2| = \left|\dfrac{5 \times -3}{13\sqrt{2} \times 5\sqrt{2}} + \dfrac{-12 \times 4}{13\sqrt{2} \times 5\sqrt{2}} + \dfrac{1 \times 1}{\sqrt{2} \times \sqrt{2}}\right| =$

$\left|\dfrac{-15}{130} - \dfrac{48}{130} + \dfrac{1}{2}\right| = \dfrac{1}{65}$ $\quad \therefore \theta = 89° \, 7' \, 6''$

EXAMPLE! (16)

find the measure of the angle between the two lines whose direction cosines are : $\left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right)$, $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

SOLUTION

$\cos\theta = \left|\frac{2\times 1}{3\times\sqrt{2}} + \frac{-2\times 1}{3\times\sqrt{2}} + \frac{1}{3}\times 0\right| = \left|\frac{2}{3\sqrt{2}} - \frac{2}{3\sqrt{2}}\right| =$ zero

$\therefore \theta = 90° \rightarrow$ the two lines are perpendicular .

The two parallel lines in space :

* if $\overrightarrow{d_1} = (a_1, b_1, c_1)$, $\overrightarrow{d_2} = (a_2, b_2, c_2)$

Are direction vectors of the lines L_1 and L_2 then $L_1 // L_2$ if and only if $\overrightarrow{d_1} // \overrightarrow{d_2}$

This condition can be satisfied by several forms:

(1) $\overrightarrow{d_1} = k\,\overrightarrow{d_2}$ (2) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (3) $\overrightarrow{d_1} \times \overrightarrow{d_2} = \overrightarrow{0}$

Remark

(1) if the two lines are parallel and there is a point belongs to one of them satisfying the equation of the other then the two lines are coincident .

(2) if L_1 , L_2 are not parallel then L_1 , L_2 are intersecting or skew

EXAMPLE! (17)

f prove that the two lines :

$\overrightarrow{r_1} = \hat{j} + t_1(\hat{i} + 2\hat{j} - \hat{k})$, $\overrightarrow{r_2} = (\hat{i} + \hat{j} + \hat{k}) + t_2(-2\,\hat{i} - 2\,\hat{j})$

SOLUTION

$\overrightarrow{d_1} = (1, 2, -1)$, $\overrightarrow{d_2} = (-2, -2, 0)$

$\frac{a_1}{a_2} = \frac{1}{-2} = -\frac{1}{2}$, $\frac{b_1}{b_2} = -\frac{2}{2} = -1$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \rightarrow \therefore$ the two lines not parallel

To find t_1 , t_2 let $\overrightarrow{r_1} = \overrightarrow{r_2} \rightarrow (0, 1, 0) + t_1(1, 2, -1)$

$= (1, 1, 1) + t_2(-2, -2, 0) = 0 + t_1 = 1 - 2t_2 \rightarrow t_1 + 2t_2 = 1 \rightarrow (1)$

$2t_1 + 2t_2 = 0 \rightarrow t_1 + t_2 = 0 \rightarrow (2)$

$0 - t_1 = 1 + 0 \quad t_1 = -1 \rightarrow (3) \quad \therefore t_2 = 1$

$\because \overrightarrow{r_1} = (0, 1, 0) + t_1(1, 2, -1) \quad \overrightarrow{r_1} = (0, 1, 0) - 1(1, 2, -1)$

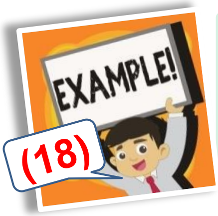
$\therefore \overrightarrow{r_1} = (-1, -1, 1)$

$\because \overrightarrow{r_2} = (1, 1, 1) + t_2(-2, 2, 0) \quad \overrightarrow{r_2} = (1, 1, 1) + 1(-2, -2, 0)$

$\therefore \overrightarrow{r_2} = (-1, -1, 1)$

The point of intersection is (- 1 , - 1, 1) the position vector of intersection point is

$\overrightarrow{r} = (-1, -1, 1)$



prove that $\overrightarrow{r_1}$ = (3, - 3, 5) + t_1 (0, - 2, 5)
$\overrightarrow{r_2}$ = (- 2, 3, 1) + t_2 (5, - 1, - 1) are perpendicular and intersect at point and find the coordinate of this point

SOLUTION

$\text{Cos } \theta = \dfrac{|\overrightarrow{d_1} \cdot \overrightarrow{d_2}|}{\|\overrightarrow{d_1}\| \, \|\overrightarrow{d_2}\|}$ $\overrightarrow{d_1} \cdot \overrightarrow{d_2}$ = (0, -5, 5). (5, -1, -1) = 0 + 5 - 5 = 0

$\text{Cos } \theta = \dfrac{0}{\|\overrightarrow{d_1}\| \, \|\overrightarrow{d_2}\|}$, $\cos \theta = 0 \therefore \theta = 90°$

$\because \dfrac{a_1}{a_2} = 0$, $\dfrac{b_1}{b_2} = \dfrac{-5}{-1} = 5$, $\dfrac{c_1}{c_2} = \dfrac{5}{-1} = -5$

$\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$ → ∴ the two lines are not parallel

To find the intersection point let $\overrightarrow{r_1} = \overrightarrow{r_2}$

→ (3, - 3, 5) + t_1 (0, - 5, 5) = (- 2, 3, 1) + t_2 (5, - 1, - 1)

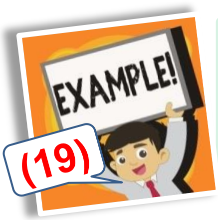
→ 3 + 0 = - 2 + 5 t_2 → 5 t_2 = 5 ∴ t_2 = 1

→ -3 - 5 t_1 = 3 - t_2 → - 5 t_2 = 3 + 3 - 1 ∴ t_1 = 1

Perpendicular lines in space :

* if $\overrightarrow{d_1} = (a_1, b_1, c_1)$, $\overrightarrow{d_2} = (a_2, b_2, c_2)$

Are direction vectors of L_1 , L_2 then $L_1 \perp L_2$ if and only if $\overrightarrow{d_1} \cdot \overrightarrow{d_2} = 0$



prove that : $\overrightarrow{r_1}$ = (1, 2, 4) + t_1 (2, - 1, 1) , $\overrightarrow{r_2}$ = (1, 1, 1) + t_2 (- 2, 7, 11)
Are perpendicular then show that the two lines are skew

SOLUTION

The direction vector of L_1 : $\overrightarrow{d_1}$ = (2, - 1, 1)

The direction vector of L_2 : $\overrightarrow{d_2}$ = (- 2, 7, 11)

$\overrightarrow{d_1} \cdot \overrightarrow{d_2}$ = (2, - 1, 1). (- 2, 7, 11) = - 4 – 7 + 11 = zero

∴ L_1 , L_2 are perpendicular let $\overrightarrow{r_1} = \overrightarrow{r_2}$

(1, 2, 4) + t_1 (2, - 1, 1) = (1, 1, 1) + t_2 (-2, 7, 11)

→ 1 + 2t_1 = 1 – 2 t_2 → $t_1 - t_2 = 0$ → (1)

→ 2 - t_1 = 1 + 7 t_2 → t_1 + 7t_2 = 1 → (2)

4 + t_1 = 1 + 11 t_2 → t_1 + 11t_2 = - 3 → (3)

Form (1) $t_1 = -t_2$ From (2) - t_2 + 7t_2 = 1 → $t_1 = \frac{1}{6}$, $t_2 = \frac{1}{6}$

∵ t_1 , t_2 not satisfying the third equation ∴ the two lines are skew.

EXAMPLE! (20)

prove that : $\overrightarrow{r_1} = (3, -1, 2) + t_1 (4, 1, 3)$, $\overrightarrow{r_2} = (0, 4, -1) + t_2 (1, -1, 2)$ are skew

SOLUTION

$\frac{a_1}{a_2} = \frac{4}{1} = 4$, $\frac{b_1}{b_2} = \frac{1}{-1} = -1$, $\frac{c_1}{c_2} = \frac{3}{2}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ ∴ the two lines not parallel

To find t_1 , t_2 let $\overrightarrow{r_1} = \overrightarrow{r_2}$

$\rightarrow (3, -1, 2) + t_1 (4, 1, 3) = (0, 4, -1) + t_2 (1, -1, 2)$

$3 + 4t_1 = t_2$ $\rightarrow 4t_1 - t_2 = -3$ $\rightarrow (1)$

$-1 + t_1 = 4 - t_2$ $\rightarrow t_1 + t_2 = 5$ $\rightarrow (2)$

$2 + 3t_1 = -1 + 2t_2$ $\rightarrow 3t_1 - 2t_2 = -3$ $\rightarrow (3)$

From (1) & (3) ∴ $t_1 = -\frac{3}{5}$, $t_2 = \frac{3}{5}$

∵ t_1 , t_2 doesn't satisfying equation (2) ∴ the two lines are skew

EXAMPLE! (21)

find the equation of the line passing through are point (2, - 1 , 3)and intersects the line $\overrightarrow{r_1} = (1, -1, 2) + t (2, 2, -1)$ orthogonally

SOLUTION

∵ $\overrightarrow{r_1} = (1, -1, 2) + t (2, 2, -1)$

∴ $c \in L_1 \rightarrow$ the point $c = (1 + 2t, -1 + 2t, 2 - t)$

The direction vector of L_2 is $\overrightarrow{d_2} = \overrightarrow{AC} = \vec{C} - \vec{A}$

$= (1 + 2t, -1 + 2t, 2 - t) - (2, -1, 3) = (-1 + 2t, -1 - t)$

∵ $L_1 \cap L_2$ orthogonally ∴ $\overrightarrow{d_1} \cdot \overrightarrow{d_2} = 0$

$(2, 2, -1) \cdot (-1 + 2t, 2t, -1 - t) = 0 \rightarrow -2 + 4t + 4t + 1 + t = 0$

$\rightarrow 9t = 1$ $\rightarrow t = \frac{1}{9}$

∴ $\overrightarrow{d_2} = \left(-1 + \frac{2}{9}, \frac{2}{9}, -1 - \frac{1}{9}\right) \rightarrow \overrightarrow{d_2} = \left(-\frac{7}{9}, \frac{2}{9}, -\frac{10}{9}\right)$

$\overrightarrow{d_2} = (-7, 2, -10)$ equation of L_2 is $\overrightarrow{r_1} = (2, -1, 3) + t(-7, 2, -10)$

EXAMPLE! (22)

Find the equation of the line passing the origin point and intersect the line $\overrightarrow{r_1}$ = (3, 1, 4) + t (2, 1, 3) orthogonally

SOLUTION

Let the intersecting point is c = (x, y, z) , c = (3 + 2t, 1 + t , 4 + 3t) and let the direction vector of L_2 is $\overrightarrow{d_2}$

$\overrightarrow{d_2} = \overrightarrow{OC} = \overrightarrow{C} - \overrightarrow{O}$ = (3+ 2t, 1 + t, 4 + 3t)

$\rightarrow \overrightarrow{d_1} \cdot \overrightarrow{d_2} = 0 \quad \rightarrow$ (2, 1, 3) . (3+ 2t, 1 + t, 4 + 3t) = 0

$\rightarrow$ 6 + 4t + 1 + t + 12 + 9 t = 0

$\rightarrow 14\,t = -19 \rightarrow t = -\dfrac{19}{14} \qquad \therefore \overrightarrow{d_2} = \left(\dfrac{2}{7}, \dfrac{5}{14}, -\dfrac{1}{14}\right)$

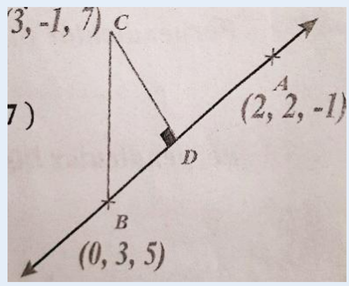
The equation of L_2 is $\overrightarrow{r_1}$ = (0, 0, 0) + t (4, 5, - 1)

The distance between a point and a straight line in space :

EXAMPLE! (23)

find the perpendicular distance between the point (3, -1, 7) and the line passing through the two points (2,2,-1) , (0, 3, 5)

SOLUTION



Let A (2,2, -1) , B(0, 3, 5) , C(3, -1, 7)

$\overrightarrow{BC} = \overrightarrow{C} - \overrightarrow{B} \rightarrow \overrightarrow{BC}$ = (3, -1, 7) - (0, 3, 5)

$\overrightarrow{BC}$ = (3, -4, 2) the direction vector of L is $\overrightarrow{d}$

$\vec{d} = \overrightarrow{AB} = \overrightarrow{A} - \overrightarrow{B}$ = (2, 2,-1) – (0, 3, 5) $\qquad \vec{d}$ = (2, -1, -6)

$\overrightarrow{BD}$ is projection of $\overrightarrow{BC}$ on the line L $(\overleftrightarrow{AB})$

$$BD = \frac{|\overrightarrow{BC} \cdot \overrightarrow{BA}|}{\|\overrightarrow{BA}\|} \rightarrow \frac{|(3,-4,2)\cdot(2,-1,-6)|}{\sqrt{4+1+36}}$$

$$BD = \frac{|6+4-12|}{\sqrt{41}} = \frac{2}{\sqrt{41}} \rightarrow BC = \sqrt{49+16+4} = \sqrt{29}$$

The perpendicular distance is CD

$$CD = \sqrt{(BC)^2 - (BD)^2} = \sqrt{29 - \frac{4}{41}} = 5.3 \text{ L.u}$$

EXAMPLE! (24)

find the perpendicular distance from the point (2, 1, - 4) on the line $\vec{r}$ = (1, -1, 2) + t (2, 3, - 2)

SOLUTION

Let $\vec{d}$ = (2, 3, - 2) , let A = (1, -1, 2) line on the line

Let B = (2, 1, - 4) $\therefore \overrightarrow{AB} = \vec{B} - \vec{A}$ = (1, 2, - 6) , $\|\overrightarrow{AB}\| = \sqrt{41}$

Projection of $\overrightarrow{AB}$ in direction of $\vec{d}$ = $\dfrac{|\overrightarrow{AB} \cdot \vec{d}\,|}{\|\vec{d}\,\|} \rightarrow \dfrac{|(1,2,-6)\cdot(2,3,-2)|}{\sqrt{4+9+4}}$

$= \dfrac{20}{\sqrt{17}}$ L.u , ∴ perpendicular Distance = $\sqrt{\left(\sqrt{41}\right)^2 - \left(\dfrac{20}{\sqrt{17}}\right)^2}$ = 4.18 L.u

EXAMPLE! (25)

prove that the perpendicular distance between the point B and the line $\vec{r}$

$\vec{r} = \vec{A} + t\,\vec{d}$ given by $\dfrac{|\overrightarrow{AB}\times\vec{d}\,|}{\|\vec{d}\,\|}$

SOLUTION

perpendicular Distance = $\|\overrightarrow{AB}\| \sin\theta \left(\times \dfrac{\|\vec{d}\|}{\|\vec{d}\|}\right)$

perpendicular Distance = $\dfrac{\|\overrightarrow{AB}\|\,\|\vec{d}\|\,\sin\theta}{\|\vec{d}\|} = \dfrac{|\overrightarrow{AB}\times\vec{d}\,|}{\|\vec{d}\,\|}$

EXERCISE 4

Answer the following question:

S.B.2017

1) find the direction cosines of the straight line with its direction ratios

(a) – 1, 2, 3 $\left[\frac{-1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right]$

(b) passes through point (3, - 1 ,5) and parallel to the vector $\overrightarrow{AB}$ where $\overrightarrow{AB}$ = (4, - 2, 2) [$\vec{r}$ = (3, - 1 ,5) + t (4, - 2, 2)]

(c) passes through the two points (3, - 1 ,0) and (0, 4 ,1) [$\vec{r}$ = (3, - 2 ,0)+ t (3, - 6, - 1)]

(d) passes through point (3, 2 ,5) and makes equal angels with the positive direction of the coordinated axes [$\vec{r}$ = (3, 2 ,5)+ t (1, 1, 1)]

S.B.2017

2) find the different forms of the equation of the straight line which:

(a) passes through point (4, - 2 ,5) and the vector $\vec{d}$ = (2, 1 ,- 1) is a direction vector of it [$\vec{r}$ = (4, - 2 ,5)+ t (2, 1, - 1)]

(b) 1, 1, 1 $\left[\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$

S.B.2017

3) write the cartesian equation for each of the following straight lines:

(a) $\vec{r}$ = (1, 3 ,9) + t (5, 4, 2) $\left[\frac{x-1}{5} = \frac{y-3}{4} = \frac{z-9}{2}\right]$

(b) the straight line passing through the point (0, 2, 0) and the vector $\vec{d}$ = (3, -1 , 4) is a direction vector of it $\left[\frac{x}{3} = \frac{y-2}{-1} = \frac{z}{4}\right]$

S.B.2017

4) if $\overrightarrow{OA} = \hat{\imath} - 2\,\hat{\jmath} + \hat{k}$, $\overrightarrow{OB}$ = - $\hat{\imath} - 3\,\hat{k}$,

$\overrightarrow{OC} = 3\,\hat{\imath} + \hat{\jmath} - 2\,\hat{k}$, $\overrightarrow{OD} = 8\,\hat{\imath} + \hat{\jmath} + 4\,\hat{k}$

Find the vector equation of the straight line which

(a) passes through the two points A, B [$\vec{r}$ = (1, - 2 ,1)+ t (- 2, 2, - 4)]

(b) passes through point D and parallel to $\overleftrightarrow{BC}$ [$\vec{r}$ = (8, 1 ,4)+ t (4, 1, 1)]

(c) passes through point C and intersects $\overleftrightarrow{AB}$ orthogonally [$\vec{r}$ = (3, 1, - 2) + t (- 19, - 11, 4)]

S.B.2017

5) find the vector equation of the straight line passing trough point A (1, - 1, 0) and parallel to the straight line passing trough the two point B (- 3, 2, 1) C (2, 1, 0) , then show tat point D (- 14, 2, 3) belongs to the straight line . [$\vec{r}$ = (1, - 1, 0) + t (5, -1 , -1)]

S.B.2017

6) find the different forms of the equation of the straight line which passes through (2, 1 , -3) and the parallel to the straight line

$\frac{x-1}{5} = \frac{y+3}{2} = \frac{1-z}{3}$ [$\vec{r}$ = (2, 1, - 3) + t (5, 2 , 3)]

S.B.2017

7) find the vector form of the equation of the straight-line

$x - 3 = \frac{y+2}{4} = \frac{2-z}{3}$ $[\vec{r} = (3, -2, 2) + t\,(1, 4, -3)]$

S.B.2017

8) find all the different forms of the equation of the straight line

$\frac{x+3}{2} = \frac{2y-1}{5} = \frac{3z+2}{4}$ $\left[\vec{r} = \left(-3, \frac{1}{2}, \frac{-2}{3}\right) + t\left(2, \frac{5}{2}, \frac{4}{3}\right)\right]$

S.B.2017

9) find the projection of point A (0, 9, 6) on the straight line passing through the two point B (1, 2, 3) C (7, - 2, 5) [(- 2, 4, 2)]

S.B.2017

10) find the distance between the point (- 2, 4, - 5) and the straight line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ $\left[\frac{\sqrt{370}}{10}\right]$

S.B.2017

11) find the equation of the straight line that passes through the point (3, -1, 0) and intersect line $\vec{r} = (2, 1, 1) + t\,(1, 2, -1)$ orthogonally. $[\vec{r} = (3, -1, 0) + t\,(-1, 1, 1)]$

S.B.2017

12) find the length of the perpendicular drawn from the point (- 4, 1, 1) to the line $\frac{x+3}{1} = \frac{y-1}{\sqrt{5}} = \frac{z+2}{2}$ $\left[\frac{\sqrt{30}}{2}\ \textit{units length}\right]$

S.B.2017

13) find the value of n which makes the two straight lines

$L_1 : \vec{r_1} = (3, -1, n) + t_1\,(4, 1, 3)$ $L_2 : x = \frac{y-4}{-1} = \frac{z+1}{2}$

Intersecting at a point, then find the point of their intersection

$\left[n = 7, \left(\frac{23}{5}, \frac{-3}{5}, \frac{41}{5}\right)\right]$

S.B.2017

14) find the measure of the angles between the two straight lines

(a) L_1: passing through the two points (- 3, 2, 4) and (2, 5, -2)

L_2: passing through the two points (1, -2, 2) and (4, 2, 3)

[60° 30°' 41'']

(b) $L_1 : \vec{r} = (2, -1, 3) + t_1\,(-1, 4, 2)$, $L_2 : \vec{r} = (0, 2, -1) + t_2\,(1, 1, 3)$

[53° 41' 23'']

(c) $L_1 : 2x = 3y = 4z$, $L_2 : \frac{x-1}{3} = \frac{y+2}{-2} = \frac{z}{-5}$ [84° 2' 20'']

(d) $L_1 : 2x = 3y - 1 = z - 3$, $L_2 : \vec{r} = (2, -1, 5) + t\,(-1, 1, 2)$

[50° 6']

S.B.2017

15) state the necessary condition (s) that make the two straight lines

$L_1 : x = x_1 + a_1 t_1, \quad y = y_1 + b_1 t_1, \quad z = z_1 + c_1 t_1$,

$L_2 : x = x_2 + a_2 t_2, , \quad y = y_2 + b_3 t_2, \quad z = z_2 + c_2 t_2$

(a) parallel $\left[\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}\right]$

(b) perpendicular [$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$]

(c) intersecting

S.B.2017

16) if the straight line $\frac{x+3}{4} = \frac{y+5}{-6}$, z =3 is parallel to the straight line $\frac{x+2}{-2} = \frac{y-3}{m}$, z = 4, find the value of m [3]

Choose the correct answer in each of the following :

S.B.2017

17) the equation of the straight line passing through the point A (- 1, 0, 2)and the vector $\vec{d} = (1, -1, 3)$ is a direction vector of it is .....

(a) $\frac{x-1}{2} = \frac{y}{1} = \frac{z-1}{-1}$ (b) $\frac{x+1}{1} = \frac{y}{-1} = \frac{z-2}{3}$

(c) $\frac{x-1}{3} = \frac{y}{-1} = \frac{z}{1}$ (d) $\frac{x-1}{1} = \frac{y-1}{-1} = \frac{z}{2}$

S.B.2017

18) the equation of the straight line passing through the point A (1, - 1, 2) and B (- 1, 0, 1) is .....

(a) $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{-1}$ (b) $\frac{x+1}{-2} = \frac{y}{1} = \frac{z+2}{-1}$

(c) $\frac{x-2}{3} = \frac{y+1}{3} = \frac{z-1}{2}$ (d) $\frac{x-1}{1} = \frac{y+1}{3} = \frac{z-2}{1}$

S.B.2017

19) if the two straight lines $\frac{x+1}{2} = \frac{y-1}{3} = \frac{z-3}{4}$ and $\frac{x}{3} = \frac{y+1}{4} = \frac{z-1}{k}$ are perpendicular, then k = ...

(a) 4 (b) – 4 (c) $\frac{9}{2}$ (d) - $\frac{9}{2}$

S.B.2017

20) if $L_1 : x = 0$, $y = z$ and $L_2 : y = 0$, $x = z$ are two straight lines in space and the measure of the angle between them is θ ,then θ = ...

(a) 60° (b) 120° (c)150° (d) 165°

S.B.2017

21) if the two straight lines $L_1 : \frac{x}{2} = \frac{y-1}{-1} = \frac{z-2}{m}$ and $L_2 : \frac{x-1}{m} = \frac{y-2}{1} = \frac{z}{-1}$ are perpendicular what is the value of m ?

(a) - 1 (b) 2 (c)1 (d) - 3

S.B.2017

22) if the straight lines $L_1 : x = 2t - 1, y = t + 1, z = t - 1$, $L_2 : x = a t - 1, y = 2t + 1, z = b t - 2$ are parallel, then a + b = ...

(a) 4 (b) - 2 (c)6 (d) 2

S.B.2017

23) the equation of the x-axis in space is ……

(a) x = 0 , y = 0 (b) x = 0, z = 0 (c) y = 0, z = 0

(d) x = 0

S.B.2017

24) which of the following points lie on the straight line $\vec{r}$ = (2, -1, 3) + t (1, 2 , - 1)?

(a) (1, 1, 1) (b) (0, 2, - 2) (c) (3, 1, 2) (d) (4, -3, 0)

Complete the following

S.B.2017

25) the vector equation of the straight line passing through point (2, -1, 3) and the vector (-1, 4, 2) is a direction vector of it is …..

S.B.2017

26) the measure of the angles between the two straight lines whose direction ratios are (1, 1, 2) and ($\sqrt{3}$ – 1, - $\sqrt{3}$ – 1, 4) equals .....

S.B.2017

27) if θ_z is the angel between the straight line through point (3, -1, 1), the origin point and the positive direction of z-axis , then cos θ_z = ….

S.B.2017

28) the direction vector the straight line passing through the two points (7, - 5, 4) and (5, -3, 3) is ….

S.B.2017

29) the measure of the angle between the straight line $\frac{x-1}{\sqrt{2}} = \frac{y-\sqrt{2}}{1} = \frac{z+1}{1}$ with the (+ve) direction of z-axis is ....

S.B.2017

30) if the straight lines $\frac{x+2}{6} = \frac{y-1}{m} = \frac{z-1}{3}$ is perpendicular to the straight line $\frac{x-9}{-2} = \frac{y+8}{1}$, z = 3 , then m = ...

S.B.2017

31) the cosine of the measure of the angle between the two straight lines $\frac{x}{1} = \frac{y}{-2} = \frac{z+1}{-2}$ and $\frac{x}{1} = \frac{y-2}{-2} = \frac{z}{2}$ equals ......

S.B.2017

32) the vector form of the equation of the straight line which passes through the point (2, -1,4) and its direction vector is $\vec{d}$ = (4, 7, 1) is ....

S.B.2017

33) if the measure of the angle between the two lines $\frac{x}{a} = \frac{y}{2} = \frac{z}{1}$ and $\frac{x}{2} = \frac{y}{1} = \frac{z}{-1}$ equals 60° , then the value of a = ...

S.B.2017

34) if the measure of the angle between the two lines $\vec{r_1}$ = (- 2, 5, -7) + t (-6, 6 , 8) and $\vec{r_2}$ = (1, -2, 3) + t' (4, 12 , - 6) equals ......

S.B.2017	35) if $\vec{A}$ = (-2, k , -3) is parallel to the straight line $\frac{x+2}{4} = \frac{y}{8} = \frac{z-1}{6}$, then k = ……..

Answer the following question:

MOE.2017	36) find the vector form of the equation of the straight line passing trough the two point A (2, - 1, 5) and B (- 3, 1, 4), then find the coordinate form of the equation the straight line [$\vec{r}$ = (2, - 1, 5) + t (-5, 2 , -1)]
S.B.2017	37) find the length of the perpendicular drawn from the point A (- 2, 3, 1) to the line $\frac{x+2}{4} = \frac{y-3}{4} = \frac{z-1}{4}$ [0]
S.B.2017	38) find the different forms of the equation of the straight line $\frac{3x+1}{5} = \frac{y-1}{2} = \frac{5-z}{3}$ $\left[\left(\frac{-1}{3}, 1, 5\right) + t\left(\frac{2}{3}, 2, -3\right)\right]$
S.B.2017	39) find the equation of the straight line which passes through the point (2, -1, 3) and intersects the straight line $\vec{r}$ = (1, -1, 2) + t (2, 2 , -1) orthogonally [(2, -1, 3) + t_2(-7, 2, - 10)]
S.B.2017	40) prove that the two straight lines L_1 : $\vec{r}$ = (4, -3, 2) + t_1 (6, 8 , -3) and L_2 : $\vec{r}$ = (1, - 1, 2) + t_2 (3, 2 , - 1) are intersecting and find the coordinates of the intersection point [(10.5, - 1)]
S.B.2017	41) find the equation of straight line passes through the origin point and cut perpendicularly the straight line $\vec{r}$ = (3, 1, 4) + k (2, 1 , 3) $\left[\frac{x}{4} = \frac{y}{-5} = \frac{z}{-1}\right]$
S.B.2017	42) discover the error: (a) the sum of squares of direction ratios for any straight line equals 1 (b) the direction cosines of the straight line passing through the two points (x_1, y_1, z_1) and (x_2, y_2, z_2) are ($x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$) (c) if (a_1 , b_1, c_1) and (a_2, b_2, c_2) are the direction ratios of the two straight lines L_1 and L_2, then the measure of the angel between them is given by the relation cos θ = $\lvert a_1a_2 + b_1b_2 + c_1c_2\rvert$

Choose the correct answer in each of the following:

S.B.2017	43) the measure of the angle between the two lines $\frac{x-3}{2} = \frac{z+1}{-2}$, y = 1 and $\frac{x+1}{1} = \frac{y-2}{2} = \frac{z+1}{-2}$ equals ……. (a) 15° (b) 30° (c)45° (d) 60°

S.B.2017	44) the equation of the straight line passing through the two points (2, -1, 3) and (0, 3, 1) is ….. (a) $\vec{r}$ = (2, -1, 3) + t (2, - 4 , 2) (b) $\vec{r}$ = (2, -1, 3) + t (2, 2 , 4) (c) $\vec{r}$ = (2, -4, 2) + t (2, - 1 , 3) (d) $\vec{r} \bullet$ (2, -4, 2) = 0
S.B.2017	45) if $L_1 : \frac{x-3}{2} = \frac{-y-1}{6} = \frac{z}{k}$ is parallel to $L_2 : \frac{x+2}{6} = \frac{y-4}{m} = \frac{z-1}{3}$, then k + m = …… (a) - 17 (b) - 10 (c)10 (d) 17
MOE 2017	46) the direction vector of the straight line L : $\frac{x-2}{3} = \frac{y+3}{2}$, z = 4equals ……. (a) (3, 2, 4) (b) (3, 2, 0) (c) (4, 2, 3) (d) (2, 3, 4)
S.B.2017	47) if $L_1 : \frac{x+2}{2} = \frac{y+3}{3} = \frac{z+5}{2}$ is perpendicular to $L_2 : \frac{x}{2} = \frac{y-5}{k} = \frac{z-6}{m}$, then 3 k + 2m = …… (a) - 1 (b) 0 (c)2 (d) 4
S.B.2017	48) the measure of the angle between the two lines x – 1= $\frac{y+2}{\sqrt{2}}$ = - z + 1 and – x = z + 3, y = 4 equals ……. (a) 45° (b) 120° (c)135° (d) 150°
S.B.2017	49) the length of the perpendicular drawn from the point A(1, 0, 2) to the straight line $\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z-3}{-2}$ equals …… (a) $\frac{\sqrt{26}}{4}$ (b) $\frac{\sqrt{26}}{5}$ (c) $\frac{\sqrt{26}}{3}$ (d) $\frac{\sqrt{26}}{6}$
MOE 2017	50) if the two straight lines $\vec{r}$ = (1, 2, 4) + t (2, - 1 , 1) and $\frac{x-1}{-2} = \frac{y-1}{7} = \frac{z-1}{m}$ are perpendicular, then m = ….. (a) 1 (b) 5 (c) 6 (d) 11
MOE 2017	51) the measure of the angle between the two straight lines y= - 1, 2 – x = z – 3 and x = 1,4 - y = z + 5 (a) 60° (b) 120° (c)90° (d) zero
MOE 2017	52) if the two straight lines $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-1}{4}$, $\frac{x}{2} = \frac{y-2}{4} = \frac{z-1}{k}$ are perpendicular, then k = ….. a) $-\frac{19}{4}$ (b)- $\frac{17}{4}$ (c) – 4.5 (d) 4.5