EXAMPLE! (8)

(a) IF $\vec{A}$ = (2, -3,0), $\vec{B}$ = (1,4,-1) find $\overrightarrow{AB}$.
(b) IF $\vec{A}$ = (1,1,-2), $\overrightarrow{AB}$ = (4,-1,2) find coordinates of the point B.

SOLUTION

(a) $\overrightarrow{AB} = \vec{B} - \vec{A}$ = (1,4,-1) - (2, -3,0) = (-1,7,-1)
(b) $\overrightarrow{AB} = \vec{B} - \vec{A}$, $\vec{B} = \overrightarrow{AB} + \vec{A}$ = (4,-1,2) + (1,1,-2) = (5,0,0)

The unit vector in the direction of a given vector:

IF $\vec{A} = (A_x, A_y, A_z) \in R^3$ Then the unit vector in the direction of $\vec{A}$ is denoted by $\overrightarrow{U_A}$, and given by the relation: $\overrightarrow{U_A} = \dfrac{\vec{A}}{\|\vec{A}\|}$

EXAMPLE! (9)

IF $\vec{A}$ = (-2,2,1), $\vec{B}$ = (3,1,–2) find the unit vector in direction of $\vec{A}$, $\vec{B}$, $\overrightarrow{AB}$

SOLUTION

$\overrightarrow{U_A} = \dfrac{\vec{A}}{\|\vec{A}\|} = \dfrac{(-2,2,1)}{\sqrt{4+4+1}} = \left(\dfrac{-2}{3}, \dfrac{2}{3}, \dfrac{1}{3}\right) \longrightarrow \overrightarrow{U_A} = \dfrac{\vec{B}}{\|\vec{B}\|} = \dfrac{(3,1,-2)}{\sqrt{9+1+4}} = \left(\dfrac{3}{\sqrt{14}}, \dfrac{1}{\sqrt{14}}, \dfrac{-2}{\sqrt{14}}\right) \longrightarrow$ $\overrightarrow{AB} = \vec{B} - \vec{A}$ = (3, 1, –2) – (–2, 2, 1) = (5, –1, –3)

$\overrightarrow{U_{AB}} = \dfrac{\overrightarrow{AB}}{\|\overrightarrow{AB}\|} = \dfrac{(5,-1,-3)}{\sqrt{25+1+9}} = \left(\dfrac{5}{\sqrt{35}}, \dfrac{-1}{\sqrt{35}}, \dfrac{-3}{\sqrt{35}}\right)$

The direction angles and direction cosines of a vector in space

IF $\vec{A} = (A_x , A_y , A_z)$ is a vector in space and IF$(\theta_x, \theta_y, \theta_z)$ are measures of the angles that the vector makes with the position direction of (x,y,z) axes respectively

$A_x = \|\vec{A}\| \cos \theta_x$, $A_y = \|\vec{A}\| \cos \theta_y$, $A_z = \|\vec{A}\| \cos \theta_z$

$\vec{A} = (A_x , A_y , A_z) = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} = \|\vec{A}\| \cos \theta_x\, \hat{\imath} + \|\vec{A}\| \cos \theta_y \hat{\jmath} + \|\vec{A}\| \cos \theta_z\, \hat{k}$

$= \|\vec{A}\| (\cos \theta_x\, \hat{\imath} + \|\vec{A}\| \cos \theta_y\, \hat{\jmath} + \|\vec{A}\| \cos \theta_z \hat{k})$

$(\theta_x, \theta_y, \theta_z)$ are direction angles of vector $\vec{A}$

$(\cos \theta_x, \cos \theta_y, \cos \theta_z)$ are direction cosines of vector $\vec{A}$

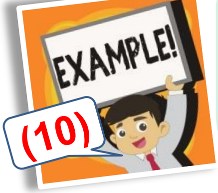
Remark

The unit vector in direction of $\vec{A}$ is

$\overrightarrow{U_A} = \cos \theta_x\, \hat{\imath} + \cos \theta_y\, \hat{\jmath} + \cos \theta_z\, \hat{k} = (\cos \theta_x , \cos \theta_y ,\cos \theta_z)$

$\|\overrightarrow{U_A}\| = \sqrt{\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z} = 1$

Where : $\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$

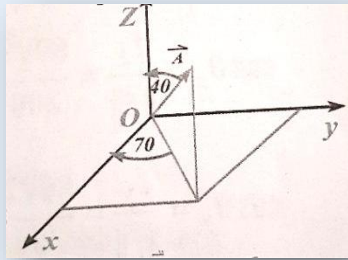


The opposite figure represents the vector $\bar{A}$ whose norm is 10 units.
(a) Express the vector A by algebraic form (cartesian components).
(b)Find measure of direction angles of vector $\bar{A}$.

SOLUTION

(a) First resolve the vector $\bar{A}$ into two components:
The first in the direction of $\overline{OZ}$ and its magnitude $A_z = \|\bar{A}\| \cos\theta_z = 10 \cos 40 = 7.66$

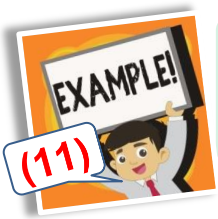
The second lies in the coordinate plane *xy*: $A_{xy} = \|\bar{A}\| \sin\theta_z = 10 \sin 40 = 6.428$
Now resolve the component A_{xy} into two components:
The first in the direction of $\overline{OX}$ and its magnitude A_x
$A_x = A_{xy} \cos 70 = 6.428 \cos 70 = 2.199$
The second in the direction of $\overline{OY}$ and its magnitude A_y
$A_y = A_{xy} \sin 70 = 6.428 \sin 70 = 6.04$
$\bar{A} = (A_x , A_y , A_z) = A_x\,\hat{\imath} + A_y\,\hat{\jmath} + A_z\,\hat{k} = (2.199, 6.04, 7.66)$



(b) $\overline{U_A} = \dfrac{\bar{A}}{\|\bar{A}\|} = \left(\dfrac{2.199}{10}, \dfrac{6.04}{10}, \dfrac{7.66}{10}\right)$

$\cos\theta_x = \dfrac{2.199}{10}$, $\therefore \theta_x = 77°17'49''$ $\quad \cos\theta_y = \dfrac{6.04}{10}$, $\therefore \theta_y = 52°50'\,35''$

$\cos\theta_z = \dfrac{7.66}{10}$, $\therefore \theta_z = 70\,,0, = 40°14''$



The opposite figure represents a force $\bar{F}$ of magnitude 200N
(a) Express the force $\bar{F}$ in the algebraic form.
(b) Find the measure of the direction angles of the force $\bar{F}$.

SOLUTION

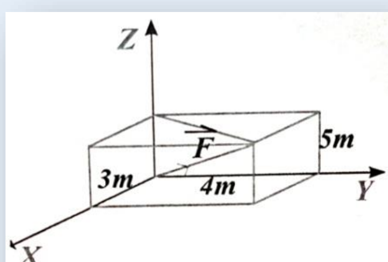
a) $\vec{f} = (\, f \cos\theta \sin\varphi \,,\, f \cos\theta \cos\varphi \,,\, f \sin\theta \,)$
$\vec{f} = (\, 200 \times \dfrac{5}{4\sqrt{2}} \times \dfrac{3}{5} \,,\, 200 \times \dfrac{5}{4\sqrt{2}} \times \dfrac{4}{5} \,,\, 200 \times \dfrac{5}{4\sqrt{2}} \,)$
$\vec{f} = (\, 60\sqrt{2} \,,\, 80\sqrt{2} \,,\, 100\sqrt{2} \,) \longrightarrow \vec{f} = 60\sqrt{2}\,\hat{\imath} + 80\sqrt{2}\,\hat{\jmath} , 100\sqrt{2}\,\hat{k}$

b) $\|\vec{f}\| = \sqrt{f_x^2 + f_y^2 + f_z^2} = \sqrt{(60\sqrt{2})^2 + (80\sqrt{2})^2 + (100\sqrt{2})^2} = 200$

$\cos\theta_x = \dfrac{f_x}{\|\vec{f}\|} = \dfrac{60\sqrt{2}}{200} \longrightarrow \theta_x = 64°53'45''$

$\cos\theta_y = \dfrac{f_x}{\|\vec{f}\|} = \dfrac{80\sqrt{2}}{200} \longrightarrow \theta_x = 55°33'$

$\cos\theta_z = \dfrac{f_x}{\|\vec{f}\|} = \dfrac{100\sqrt{2}}{200} \longrightarrow \theta_z = 45°$



EXERCISE 2

S.B.2017 — 1) find the norm of each of the following vectors :

(a) $\vec{A} = (2, -1, 0)$ (b) $\vec{B} = (1, 2, -2)$

(c) $\vec{C} = \hat{j}$ (d) $\vec{D} = \hat{\imath} - 4\hat{j}$

S.B.2017 — 2) if $\vec{A} = (1, -3, 2)$ and $\vec{B} = (0, 2, 3)$ find $\|\vec{A}\|$ and $\|\vec{A} + \vec{B}\|$ [$\sqrt{14}$, $3\sqrt{3}$]

S.B.2017 — 3) A = (- 2, 3, 5) and B = (1, 4, - 2) , find $\overrightarrow{AB}$ [(3, 1, - 7)]

S.B.2017 — 4) if $\vec{C} = (1, -2, 2)$, find the unit vector in the direction of $\vec{C}$ $\left[\left(\frac{1}{3}, \frac{-2}{3}, \frac{2}{3}\right)\right]$

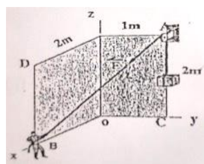
S.B.2017 — 5) If the vector A makes with the positive direction of the coordinate axes x, y and z angles of measure 60° 80° and θ, where θ is an acute angle,

(a) Find the measure of θ. [31.96°]

(b) Find if $\vec{A}$ if $\|\vec{A}\| = 13$ [$6.5\,\hat{\imath} + 2.26\,\hat{j} + 11.03\hat{k}$]

S.B.2017 — 6) If the tension force in a string equals 21 Newton,

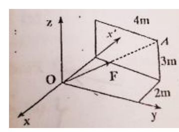
Find the components of the force $\vec{F}$.



[(14, - 7, - 14)]

S.B.2017 — 7) Find the vector $\vec{A}$ whose norm is $21\sqrt{3}$ and makes equal angles with the positive directions of the coordinate axes. [$\pm 21\,(\hat{\imath} + \hat{j} + \hat{k})$]

S.B.2017 — 8) Use the opposite figure to find the components of the force F whose magnitude is $12\sqrt{29}$ Newton in the direction of the coordinate axes. [(-24, 48,36)]



S.B.2017 — 9) Find the direction angles of the vector $\vec{C}$ $3\,\hat{\imath} - 4\,\hat{j} + 5\,\hat{k}$ with the positive direction of the coordinate axes. [64.9°, 124.48°, 45°]

Choose the correct answer in each of the following:

S.B.2017 — 10) if $\vec{A} = (-2, k, 1)$ and $\|\vec{A}\| = 3$ units, then k = ……

(a) 4 (b) - 4 (c) ±2 (d) $\sqrt{14}$

S.B.2017 — 11) if 30°, 70° , θ are the direction angles of a vector , then one of the values of θ = ……

(a) 100° (b) 80° (c) 260° (d) 68.61°

S.B.2017

12) if $\vec{A} = (-1, 5, -2)$, $\vec{B} = (3, 1, 1)$ and $\vec{A} + \vec{B} + \vec{C} = \hat{\imath}$, then $\vec{C} = \ldots\ldots$

(a) $\hat{\imath} + 6\hat{\jmath} - \hat{k}$ (b) $-\hat{\imath} - 6\hat{\jmath} + \hat{k}$

(c) $\hat{\imath} + 4\hat{\jmath} - 3\hat{k}$ (d) $\hat{\imath} + 4\hat{\jmath} - \hat{k}$

S.B.2017

13) Which of following vector is a unit vector?

(a) $(-3, 2, 2)$ (b) $\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right)$

(c) $\left(\frac{3}{5}, \frac{-4}{5}, 0\right)$ (d) $\left(\frac{-1}{12}, \frac{-3}{12}, \frac{2}{12}\right)$

S.B.2017

12) if ℓ, m and n are the direction cosines of vector $\vec{A}$, then ……

(a) $\ell + m + n = 1$ (b) $\ell = m = n$

(c) $\ell^2 + m^2 + n^2 = 1$ (d) $\ell + m + n = \|\vec{A}\|$

S.B.2017

13) the direction cosines of the vector $\vec{A} = (-2, 1, 2)$ is ……..

(a) $(-2, 1, 2)$ (b) $\left(\frac{-2}{3}, \frac{1}{3}, \frac{2}{3}\right)$

(c) $\left(\frac{-5}{2}, 5, \frac{5}{5}\right)$ (d) $(-1, 1, 1)$

S.B.2017

14) if $\vec{A} = (3, -2, m)$ and $\|\vec{A}\| = \sqrt{22}$, then m = ……

(a) 21 (b) ± 9 (c) ± 3 (d) 17

S.B.2017

15) if $\vec{A} = 4\hat{\imath} - 3\hat{\jmath} + 5\hat{k}$, then the component of $\vec{A}$ in the direction of y-axis equals ……

(a) 4 (b) 3 (c) - 3 (d) 5

S.B.2017

16) if the direction angles of a vector are 45°, 45°, θ, then $\theta = \ldots.$

(a) 45° (b) 90° (c) 0 (d) 60°

S.B.2017

17) if $\vec{A} = (-2, 4, 6)$ and $\vec{B} = (0, k, 3)$, where $k \in z^+$ and $\|\overrightarrow{AB}\| = 7$, then the value of k = ……..

(a) 10 (b) 8 (c) 6 (d) 4

S.B.2017

18) if A (-4, -2, 3), B (1, 2, K), and the length of AB = $\sqrt{77}$, then one of the value of k is ……..

(a) 2 (b) 4 (c) 6 (d) 9

S.B.2017

19) if $\vec{A} = (-1, 3, 4)$ and $\vec{B} = (0, -2, 5)$, then $\|\overrightarrow{AB}\| = \ldots\ldots$

(a) $2\sqrt{3}$ (b) $3\sqrt{3}$ (c) $4\sqrt{3}$ (d) $5\sqrt{3}$

S.B.2017

20) if $\vec{A}$ = (1, - 1, 2) , $\vec{B}$ = (0, 2, - 3) and $\vec{C}$ = (-2, 1, 0) then $\|3\vec{A} - \vec{B} + \vec{C}\|$ = ……..

(a) $8\sqrt{3}$ (b) 11 (c) 12 (d) $7\sqrt{2}$

S.B.2017

21) the direction cosines of the vector (2, - 4, 4) are ……

(a) (2, - 4, 4) (b) (1, - 2, 2) (c) $\left(\frac{1}{3}, \frac{-2}{3}, \frac{2}{3}\right)$ (d) $\left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$

S.B.2017

22) if $\overrightarrow{AB} = 3\,\hat{\imath} + 3\,\hat{\jmath} + 7\,\hat{k}$ and $\overrightarrow{BC} = \hat{\jmath} + 5\,\hat{k}$, then $\|\overrightarrow{AC}\|$ = ……..

(a) 13 (b) 12 (c) 10 (d) 9

S.B.2017

23) if $\vec{A}$ = (-7, 3, 10) and $\vec{B}$ = (- 4, - 1, - 2), then the unit vector in the direction of $\overrightarrow{AB}$ = ……..

(a) $\left(\frac{3}{13}, \frac{4}{13}, \frac{12}{13}\right)$ (b) $\left(\frac{3}{13}, \frac{-4}{13}, \frac{-12}{13}\right)$ (c) $\left(\frac{-3}{13}, \frac{4}{13}, \frac{12}{13}\right)$ (d) $\left(\frac{-3}{13}, \frac{-4}{13}, \frac{-12}{13}\right)$

S.B.2017

24) if A (-2, 0, 3) and B (4, 2,- 5), then $\|\overrightarrow{AB}\|$ = …….. units of length

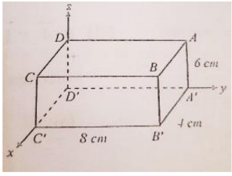
(a) $\sqrt{12}$ (b) $\sqrt{40}$ (c) $\sqrt{44}$ (d) $\sqrt{104}$

S.B.2017

25) in the opposite figure;

ABCD A' B' C' D' is a cuboid in which

A (4, 0, 0), C (0, 9, 0), D' (0, 0, 7), then $\|\overrightarrow{AC'}\|$ = ……..



(a) $\sqrt{146}$ (b) $\sqrt{144}$ (c) 5 (d) $\sqrt{20}$

MOE 2017

26) if 30°, 70° , θ° are the direction angles of a vector , then one of the values of θ = ……

(a) 0° (b) 30° (c) 60° (d) 90°

MOE 2017

27) if A (2, 1, 0) and B (1, 1, 0), then the unit vector of $\overrightarrow{AB}$ is ……

(a) $\hat{\imath}$ (b) $\hat{\jmath}$ (c) $\hat{k}$ (d) - $\hat{\imath}$

S.B.2017

28) if the direction angles of a vector are 45° , 45° , θ , then θ = ….

(a) 45° (b) 90° (c) 0 (d) 60°

Answer the following:

S.B.2017

29) find the direction cosines of a straight line which makes acute equals angles with the coordinates $\left[\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)\right]$

S.B.2017

30) if θ_x , θ_y , θ_z are direction angles of a line in a space, prove that $\sin^2\theta_x + \sin^2\theta_y + \sin^2\theta_z = 2$

S.B.2017 | 40) find the vector $\vec{A}$ if its norm is 5 units and makes equal angles with the positive direction the cartesian axes $\left[\pm\frac{5\sqrt{3}}{3}\left(\hat{\imath}+\hat{\jmath}+\hat{k}\right)\right]$

S.B.2017 | 45) ABCD is a parallelogram where A (- 1, 2, 1), B (2, 0, - 1) and C (3, 1, 4), find, using vectors , the coordinates of the point D. [(0, 3, 6)]

S.B.2017 | 46) find the coordinates of the point A whose distance from the origin is 8 units of length, such that the straight line $\overleftrightarrow{AO}$ makes with the two axes $\overleftrightarrow{zz}'$, $\overleftrightarrow{xx}'$ angles of measures $\frac{\pi}{4}$, $\frac{\pi}{3}$ respectively.[(4, 4, $4\sqrt{2}$), (4, - 4, $4\sqrt{2}$)]

Complete the following:

S.B.2017 | 47) the unit vector in the direction of $\vec{A}$ = (2, 3, $2\sqrt{3}$) equals ……

S.B.2017 | 48) if $\vec{A}$ = (1, 2, - 4) , $\vec{B}$ = (1, 1, k - 1) $\|\vec{A}+\vec{B}\|$ = 7 units of length , then k = ……..

S.B.2017 | 49) if $\vec{A} = \left(-\frac{1}{2}, \frac{3}{4}, k\right)$ is a unit vector, then the value of k = …… or ….

Vector Multiplication

There are two kinds of vectors multiplication :

(1) The scalar product (the dot product)

The vector product (the cross product)

Scalar product of two vectors (Dot Product)

* If $\vec{A}, \vec{B}$ are two vectors the measure of the minor angles between them

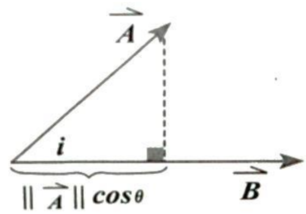
is θ the scalar product of the two vectors $\vec{A}, \vec{B}$ is $\vec{A}, \vec{B}$

$\vec{A} \cdot \vec{B} = \|\vec{A}\| \times \|\vec{B}\| \cos\theta \rightarrow \vec{A} \cdot \vec{B} = AB\cos\theta$

* The absolute value of the scalar product equals the area of the rectangle whose dimensions are the norm of one of the two vectors

$\left(\|\vec{B}\|\right)$ and the component of the other on it $\left(\|\vec{A}\| \cos\theta\right)$

Properties of scalar product :



1) Commutative property : $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

2) $\vec{A} \cdot \vec{A} = \|\vec{A}\|^2$

3) $\vec{A} \cdot \vec{B}$ = zero if and only if $\vec{A} \perp \vec{B}$

4) Distributive property: $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$

5) If K is a scalar (real number) then, $(K\vec{A}) \cdot \vec{B} = \vec{A}\,(k\,\vec{B}) = k\,(\vec{A} \cdot \vec{B})$

Remark

1) The two vectors must be outwards of the same point or inwards to the same point

2) $\theta \in [0, \pi]$

EXAMPLE! (1)

if $\|\vec{A}\| = 2\,\|\vec{B}\| = 8$, $\theta = 60°$, Find $\vec{A} \cdot \vec{B}$

SOLUTION

$\|\vec{A}\| = 8$, & $2\,\|\vec{B}\| = 8 \rightarrow \|\vec{B}\| = 4 \rightarrow \vec{A} \cdot \vec{B} = AB\cos\theta = 8 \times 4 \times \cos 60 = 16$

EXAMPLE! (2)

If $\vec{A}$. $\vec{B}$ are two vectors the angle between them 135° and if $\|\vec{A}\|$ = 6, $\|\vec{B}\|$ = 10, Find $\vec{A}$. $\vec{B}$

SOLUTION

$\vec{A}$. $\vec{B}$ = AB cos 0 = 6 × 10 × cos 135 = -30$\sqrt{2}$

Remark

If $\vec{A} \perp \vec{B}$ then $\vec{A}$. $\vec{B}$ = zero * If $\vec{A}$. $\vec{B}$= zero then $\vec{A} \perp \vec{B}$

EXAMPLE! (3)

If ($\hat{\imath}$, $\hat{\jmath}$,$\hat{k}$) are three unit vectors of righ hand system. Find $\hat{\imath}$. $\hat{\imath}$, $\hat{\jmath}$. $\hat{\jmath}$, $\hat{k}$. $\hat{k}$

SOLUTION

$\hat{\imath}$. $\hat{\imath}$ = $\|\hat{\imath}\|$ $\|\hat{\imath}\|$ cos θ = 1 × 1×1 = 1 → $\hat{\jmath}$. $\hat{\jmath}$ = 1×1×1 = 1 → $\hat{k}$. $\hat{k}$ = 1

EXAMPLE! (4)

if ($\hat{\imath}$, $\hat{\jmath}$,$\hat{k}$) are three unit vectors of right hand system. Find $\hat{\imath} \cdot \hat{\imath}$, $\hat{\jmath}$. $\hat{\jmath}$, $\hat{k}$. $\hat{k}$. $\hat{\imath}$

SOLUTION

$\hat{\imath}$. $\hat{\jmath}$ = $\|\hat{\imath}\|$ $\|\hat{\jmath}\|$ cos 90 = 1 × 1 × 0 = zero ∵ $\hat{\jmath}$. $\hat{k}$ = zero → $\hat{k}$. $\hat{\imath}$ = zero

EXAMPLE! (5)

ABCD is a square of side length 10cm find each of the following:

a) $\overrightarrow{AB}.\overrightarrow{DC}$. b) $\overrightarrow{AB} \cdot \overrightarrow{BC}$. c) $\vec{B}.\overrightarrow{CA}$

SOLUTION

a) $\overrightarrow{AB}.\overrightarrow{DC}$ = $\|\overrightarrow{AB}\|$ $\|\overrightarrow{DC}\|$ COS 0 = 10 × 10 ×1 = 100

b) $\overrightarrow{AB} \cdot \overrightarrow{BC}$ = $\|\overrightarrow{AB}\|$ $\|\overrightarrow{BC}\|$ cos 90 = zero

c) $\overrightarrow{AB}$. $\overrightarrow{CA}$ = $\|\overrightarrow{AB}\|$ $\|\overrightarrow{CA}\|$ Cos 135 = 10 × 10$\sqrt{2}$ × -$\frac{1}{\sqrt{2}}$ = -100

another solution: $\overrightarrow{AB} \cdot \overrightarrow{CA}$ = $\overrightarrow{AB}$:(-$\overrightarrow{AC}$) = -$\overrightarrow{AB} \cdot \overrightarrow{AC}$ = - $\|\overrightarrow{AB}\|$ $\|\overrightarrow{AC}\|$ cos 45

= -10 × 10 $\sqrt{2}$ × $\frac{1}{\sqrt{2}}$ = -100

EXAMPLE! (6)

ABC is equilateral triangle of side length 8cm find the following:

a) $\overrightarrow{AB}.\ \overrightarrow{AC}$, b) $\overrightarrow{AB}\cdot\overrightarrow{BC}$, c) $2(\overrightarrow{AB})\ .\ 3(\overrightarrow{BC})$

SOLUTION

(a) $\overrightarrow{AB}.\ \overrightarrow{AC} = \|\overrightarrow{AB}\|\ \ \|\overrightarrow{AC}\|\ \ \text{Cos } 60 = 10 \times 10 \times \frac{1}{2} = 50$

(b) $\overrightarrow{AB}.\ \overrightarrow{BC} = -\ (\overrightarrow{BA}.\ \overrightarrow{BC}) = -\ (\|\overrightarrow{BA}\|\ \ \|\overrightarrow{BC}\|\ \cos 60) = -\ 10 \times 10 \times \cos 60 = -\ 50$

(c) $2\ (\overrightarrow{AB})\ .\ 3\ (\overrightarrow{BC}) = 6\ (\overrightarrow{AB}.\ \overrightarrow{BC}) = 6 \times -\ 50 = -\ 300$

The scalar product of two vectors in orthogonal coordinate system,

If $\vec{A} = (A_x , A_y , A_z)$, $\vec{B} = (B_x , B_y , B_z)$, then $\vec{A}.\ \vec{B} = (A_x , A_y , A_z)\ .\ (B_x , B_y , B_z)$

$\vec{A}.\ \vec{B} = A_x B_x + A_y B_y + A_z B_z$

Proof

Let $A = (A_x , A_y , A_z) \therefore \vec{A} = A_x\ \hat{\imath} + A_y\ \hat{\jmath} + A_z\ \hat{k}$

$\vec{B} = (B_x , B_y , B_z) \therefore \vec{B} = B_x\ \hat{\imath} + B_y\ \hat{\jmath} + B_z\ \hat{k}$

$\vec{A}.\ \vec{B} = (A_x\ \hat{\imath} + A_y\ \hat{\jmath} + A_z\ \hat{k})\ .\ (B_x\ \hat{\imath} + B_y\ \hat{\jmath} + B_z\ \hat{k})$

Since , $\hat{\imath}\ .\ \hat{\imath} = \hat{\jmath}\ .\ \hat{\jmath} = \hat{k}\ .\ \hat{k} = 1 \quad \rightarrow \hat{\imath}\ .\ \hat{\jmath} = \hat{\jmath}.\ \hat{k} = \hat{k}\ .\ \hat{\imath} = 0$

$\therefore \vec{A}.\ \vec{B} = A_x\ B_x + A_y\ B_y + A_z\ B_z$

Remark

If $\vec{A} = (A_x\ A_y)$, $\vec{B} = (B_x , B_y) \rightarrow \vec{A}.\ \vec{B} = (A_x\ A_y)\ .\ (B_x , B_y)$

$\vec{A}.\ \vec{B} = A_x\ B_x + A_y\ B_y$

EXAMPLE! (7)

If $\vec{A} = 2\ \hat{\imath} + 3\ \hat{\jmath} + 4\ \hat{k}$, $\vec{B} = -\ \hat{\imath} - 2\ \hat{\jmath} + \hat{k}$. find $\vec{A}.\ \vec{B}$

SOLUTION

$\vec{A}.\ \vec{B} = (2, 3, 4)\ .\ (-\ 1, -\ 2, 1) = -\ 2 - 6 + 4 = -\ 4$

EXAMPLE! (8)

find $\vec{A}.\ \vec{B}$ in each of the following :

a) $\vec{A} = (-\ 1, 3, 2)$, $\vec{B} = (4, -\ 2, 5)$ and what do you deduce ?

b) $\vec{A} = (2, -1)$, $\vec{B} = (-\ 3, 1)$

SOLUTION

a) $\vec{A}.\ \vec{B} = (-\ 1, 3, 2)\ .\ (4, -\ 2, 5) \rightarrow \vec{A}.\ \vec{B} = -\ 4 - 6 + 10 = \text{zero} \rightarrow \vec{A} \perp \vec{B}$

b) $\vec{A}.\ \vec{B} = (2, -1)\ .\ (-\ 3, 1) = -\ 6 - 1 = -\ 7$

**** The angle between two vector :***

$\vec{A}.\vec{B} = \|\vec{A}\| \; \|\vec{B}\| \cos\theta$

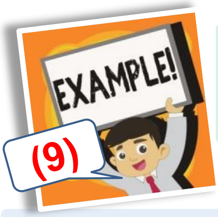
θ : is the measure of the minor angle between the two non zeros vectors $\vec{A}, \vec{B}$ where $\theta \in [0, \pi]$ $\cos\theta =$

Special cases :

(1) if $\cos\theta = 1$ then $\vec{A} \,//\, \vec{B}$ in the same direction

(2) if $\cos\theta = -1$ then $\vec{A} \,//\, \vec{B}$ in the opposite direction

(3) if $\cos\theta =$ zero, then $\vec{A} \perp \vec{B}$

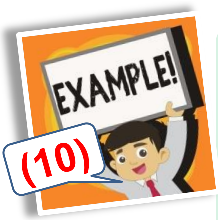


Find the measure of the angle between the two vectors $\vec{A} = (4,3,7)$, $\vec{B} = (2,5,4)$

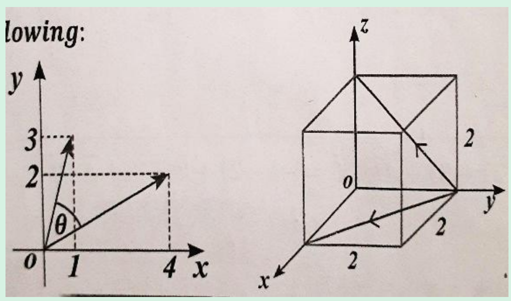
SOLUTION

$\vec{A}.\vec{B} = \|\vec{A}\| \; \|\vec{B}\| \cos\theta \rightarrow \cos\theta = \dfrac{\vec{A}.\vec{B}}{\|\vec{A}\|\|\vec{B}\|} \rightarrow \dfrac{(4,3,7).(2,5,4)}{\sqrt{74}\sqrt{45}} = \dfrac{51}{\sqrt{74}\sqrt{45}}$

$\theta = 27° \, 53' \, 50''$



Find θ in each of the following:



SOLUTION

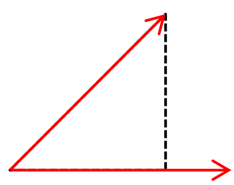
a) let $\vec{A} = (4,2)$, $\vec{B} = (1,3) \rightarrow \cos\theta = \dfrac{\vec{A}.\vec{B}}{\|\vec{A}\|\|\vec{B}\|} = \dfrac{4+6}{2\sqrt{5} \times \sqrt{10}} = \dfrac{\sqrt{2}}{2} \rightarrow \theta = 45°$

b) let $\vec{A} = (2,-2,0)$, $\vec{B} = (0 - 2,2) - \cos\theta = \dfrac{\vec{A}.\vec{B}}{\|\vec{A}\|\|\vec{B}\|} = \dfrac{0+4+0}{2\sqrt{2} \times 2\sqrt{2}} = \dfrac{1}{2}$

$\therefore \theta = 60°$

Component (projection) of a vector in a direction of another vector.

* If $\vec{A}$, $\vec{B}$ are two vectors then the component (projection) of vector $\vec{A}$ in direction vector $\vec{B}$ (denoted by A_B). $A_B = \|\vec{A}\| \cos\theta \rightarrow A_B = \dfrac{\vec{A}.\vec{B}}{\|\vec{B}\|}$



EXAMPLE! (11)

Find the component of the force $\vec{f}$ = (2,-3,5) in direction of $\overrightarrow{AB}$ where A(1,4,0), B(-1,2,3)

SOLUTION

$\overrightarrow{AB} = \vec{B} - \vec{A} \rightarrow \overrightarrow{AB}$ = (-1,2,3) - (1,4,0) = (-2,-2,3)

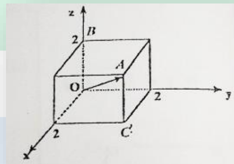
AB = $\sqrt{4 + 4 + 9} = \sqrt{17}$,

$\vec{f} \cdot \overrightarrow{AB}$ = (2, -3,5) . (-2,-2,3) = - 4 + 6 + 15 = 17 units

$F_{AB} = \dfrac{\vec{F} \cdot \overrightarrow{AB}}{\|\overrightarrow{AB}\|} \rightarrow F_{AB} = \dfrac{17}{\sqrt{17}} * \dfrac{\sqrt{17}}{\sqrt{17}} = \sqrt{17}$ unit force

EXAMPLE! (12)

The opposite figure represents a cube the length of its edge 2 L.U, find projection of $\overrightarrow{OA}$ in direction of $\overrightarrow{CB}$.



SOLUTION

$\overrightarrow{OA}$ = (2,2,2) , $\overrightarrow{CB} = \vec{B} - \vec{C}$ = (0,0,2) - (2,2,0) = (-2,-2, 2)

the projection of $\overrightarrow{OA}$ in direction of $\overrightarrow{CB} \rightarrow \dfrac{\overrightarrow{OC} \cdot \overrightarrow{CB}}{\|\overrightarrow{CB}\|} = \dfrac{-4-4+4}{2\sqrt{3}} = -\dfrac{2\sqrt{3}}{3}$

Note

The component of a vector in a direction of another vector vanishes

a) if the two vectors are perpendicular (θ = 90°)

b) if one or both of the two vectors is the zero vector $\vec{O}$

Using the scalar product to find the work done by a force:

* If a force acted on a body and moved it a displacement S then we say the force exerted a work. The work is given by the relation

$W = \vec{F} \cdot \vec{S} \rightarrow W = \|\vec{F}\| \, \|\vec{S}\| \cos\theta \rightarrow W = F\, S \cos\theta$

unit of measuring the work = units of force × unit of displacement.

EXAMPLE! (13)

The force $\vec{F} = \hat{\imath} - 2\,\hat{\jmath} + 3\,\hat{k}$ Newton acts upon a body and moved from the point A (-3,1, 0) to the point B (2,0, -2) Find the work done by the force $\vec{f}$ where the displacement is measured by meter.

SOLUTION

$\overrightarrow{S} = \overrightarrow{AB} \rightarrow \vec{S} = \vec{B} - \vec{A} \rightarrow \vec{S} = (2,0,-2) - (-3,1,0) = (5,-1,-2)$

$W = \vec{F}.\vec{S} \rightarrow W = (1,-2,3).(5,-1,-2) = 5 + 2 - 6 = 1$ N.m = 1 Joule

EXAMPLE! (14)

A body moves under the action of the force $\vec{F} = -6\hat{\imath} + 8\,\hat{\jmath}$ from the point A(-1,3) to the point B (4,7)find the work done by the force $\vec{F}$.

SOLUTION

$W = \vec{F}.\vec{S}$, $\vec{S} = \overrightarrow{AB} = B - A = (4,7) - (-1,3) = (5,4)$

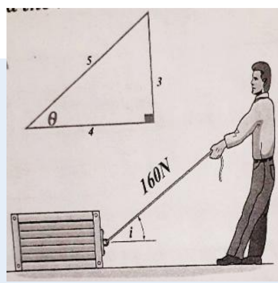
$W = (-6,8).(5,4)$ $W = -30 + 32 = 2$ units of work

EXAMPLE! (15)

In the opposite figure :

A man draw a box by a tension force with magnitude 160 newton and inclines to the horizontal at angle whose tangent is $\frac{3}{4}$ to move it horizontally a distance 5m. find the work done by this force.

SOLUTION



Work = $\vec{F}.\vec{S}$

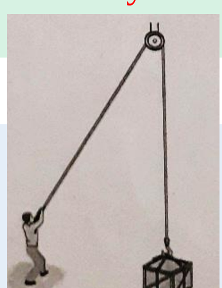
$W = \|\vec{F}\|\,\|\vec{S}\|\,\cos\theta$

$W = 160 \times 5 \times \frac{4}{5} = 640$ joule

EXAMPLE! (16)

In the opposite figure: A man lift a box by a string passing over a smooth pulley and inclines to the vertical at angle of measure 30°. If the tension at the string equal 120 newton to raise the box a distance 3 meters from the ground surface. Find the work done by the tension force.

SOLUTION



Work = $\vec{T}.\vec{S}$

$W = \|\vec{T}\|\,\|\vec{S}\|\,\cos\theta$

$W = 120 \times 3 \times \cos 150 = -311.78$ joule

The Vector Product

(cross product) of two vectors

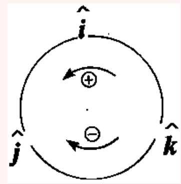
If $\vec{A}$, $\vec{B}$ are two non zero vectors in a plane including an angle of measure . Then $\vec{A} \times \vec{B} = (\|\vec{A}\| \, \|\vec{B}\| \sin \theta)\, \hat{c}$

$\hat{c}$: is a unit vector perpendicular to the plane containing $\vec{A}$, $\vec{B}$.

Right Hand Rule:

The direction of the unit vector $\hat{c}$ is defined (up or down) according to the right hand rule where the curved fingers of the right hand show the direction of the relation from $\vec{A}$ to $\vec{B}$,then the thumb shows the direction of the vector $\hat{c}$.

Remark



1) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$ by applying the right hand rule.

2) For any vector $\vec{A}$, $\vec{A} \times \vec{A} = \vec{0}$

3) By applying the right hand rule on the set of orthogonal unit vectors then

$\hat{\imath} \times \hat{\jmath} = \hat{k}$, $\hat{\jmath} \times \hat{k} = \hat{\imath}$, $\hat{k} \times \hat{\imath} = \hat{\jmath}$, $\hat{\imath} \times \hat{k} = -\hat{\jmath}$, $\hat{k} \times \hat{\jmath} = -\hat{\imath}$, $\hat{\jmath} \times \hat{\imath} = -\hat{k}$

EXAMPLE! (17)

$\vec{A}$, $\vec{B}$ are two vectors in a plane the measure of the angle between them 70° if $\|\vec{A}\| = 15$, $\|\vec{B}\| = 17.5$ Find norm of $\vec{A} \times \vec{B}$.

SOLUTION

$\vec{A} \times \vec{B} = (\|\vec{A}\| \, \|\vec{B}\| \sin \theta)\, \hat{c}$

$\|\vec{A} \times \vec{B}\| = \|AB \sin \theta \, \hat{c}\| = AB \sin \theta = 15 \text{ x } 17.5 \times \sin 70 = 246.67$

EXAMPLE! (18)

If $\vec{A} \times \vec{B} = -65\hat{c}$ and $\|\vec{A}\| = 5$, $\|\vec{B}\| = 26$ Find the measure of the angle between the two vectors $\vec{A}$, $\vec{B}$

SOLUTION

$\vec{A} \times \vec{B} = (\|\vec{A}\| \, \|\vec{B}\| \sin \theta)\, \hat{c} \longrightarrow -65\, \hat{c} = 5 \times 26 \sin \theta \, \hat{c}$

$-65 = 130 \sin \theta \longrightarrow \sin \theta = -\frac{1}{2} \longrightarrow \theta = 210° \text{ or } 330°$

The cross product in the cartesian coordinate

* if $\vec{A} = (A_x , A_y , A_z)$, $\vec{B} = (B_x , B_y , B_z)$, then

$\vec{A} \times \vec{B} = (A_x\ \hat{\imath} + A_y + A_z\ \hat{k})\ .\ (B_x\ \hat{\imath} + B_y\ \hat{\jmath} + B_z\ \hat{k})$

$= (A_x B_z - A_z B_y)\ \hat{\imath}\ -\ (A_x B_z - B_x A_z)\ \hat{\jmath} + (A_x B_y - A_y B_x)\ \hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_x B_z - A_z B_y)\ \hat{\imath}\ -\ (A_x B_z - B_x A_z)\ \hat{\jmath} + (A_x B_y - A_y B_x)\ \hat{k}$$

If $\vec{A} = (A_x , A_y)$, $\vec{B} = (B_x , B_y)$ $\therefore \vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & 0 \\ B_x & B_y & 0 \end{vmatrix} = (A_x B_y - A_y B_x)\ \hat{k}$

EXAMPLE! (19)

If $\vec{A}$ (- 2, 3, 1) , $\vec{B}$ (1, 2, 4) , find $\vec{A} \times \vec{B}$ then deduce the unit vector perpendicular to the plane containing the two vectors $\vec{A}$, $\vec{B}$

SOLUTION

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2 & 3 & 1 \\ 1 & 2 & 4 \end{vmatrix} = (3 \times 4 - 2 \times 1)\ \hat{\imath} - (-2 \times 4 - 1 \times 1)\ \hat{\jmath} + (-2 \times 2 - 1 \times 3)\ \hat{k}$$

$\vec{A} \times \vec{B} = 10\ \hat{\imath} + 9\ \hat{\jmath} - 7\ \hat{k}$

The required unit vector $\hat{c} = \dfrac{\vec{A} \times \vec{B}}{\|\vec{A} \times \vec{B}\|} = \dfrac{\vec{A} \times \vec{B}}{\|\vec{A}\|\|\vec{B}\| \sin\theta}$

$\hat{c} = \dfrac{10\ \hat{\imath} + 9\ \hat{\jmath} - 7\ \hat{k}}{\sqrt{100+81+49}}$ $\qquad \hat{c} = \dfrac{10}{\sqrt{230}}\ \hat{\imath} + \dfrac{9}{\sqrt{230}}\ \hat{\jmath} - \dfrac{7}{\sqrt{230}}\ \hat{k}$

EXAMPLE! (20)

if $\|\vec{A}\| = 6$ and the direction cosines of vector $\vec{A}$ are

Respectively $\frac{2}{3}$,- $\frac{2}{3}$, $\frac{1}{3}$ and the vector $\vec{B} = (-2, 3, 5)$, find $\vec{A} \times \vec{B}$

SOLUTION

$\vec{A} = (A \cos\theta_x, A \cos\theta_y, A \cos\theta_z) = (6 \times \frac{2}{3} , 6 \times -\frac{2}{3} , 6 \times \frac{1}{3}) = (4,-4,2)$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & -4 & 2 \\ -3 & 3 & 5 \end{vmatrix} = -26\hat{\imath} - 24\ \hat{\jmath} + 4\ \hat{k}$$

Properties of cross product:

* If $\vec{A}$, $\vec{B}$ are two vectors, the measure of the angle between them is θ then

1) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$ (not commutative)

2) $\vec{A} \times \vec{A} = \vec{B} \times \vec{B} = \vec{0}$

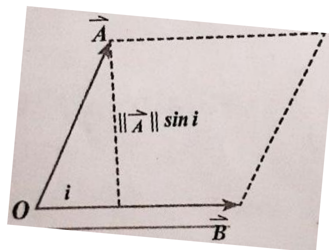
3) If $\vec{A} \times \vec{B} = \vec{0}$ then : $\vec{A}$ // $\vec{B}$ or one of the two vectors equal $\vec{0}$

4) $\vec{A} \times (\vec{B} + \vec{C}) = (\vec{A} \times \vec{B}) + (\vec{A} \times \vec{C})$ (distributive property)

5) $(K\vec{A}) \times \vec{B} = \vec{A} \times (K\vec{B}) = K(\vec{A} \times \vec{B})$, where K is a scalar (real number)

**** Parallelism of two vectors:*** $\vec{A}$ // $\vec{B}$ $(\longleftrightarrow)\vec{A} \times \vec{B} = \vec{0}$

$(A_y B_z - A_z B_y)\,\hat{\imath} - (A_x B_z - B_x A_y)\,\hat{\jmath} + (A_x B_y - A_y B_x)\,\hat{k} = \vec{0}$



$A_y B_z = A_z B_y \rightarrow \frac{A_y}{B_y} = \frac{A_z}{B_z} \rightarrow A_x B_z = B_x A_z \rightarrow \frac{A_x}{B_x} = \frac{A_z}{B_z}$

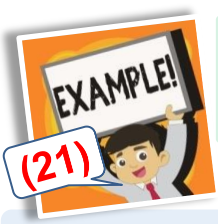
$A_x B_y = A_y B_x \rightarrow \frac{A_x}{B_x} = \frac{A_y}{B_y} \rightarrow \therefore \frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}$

Let $\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z} = K$ $\rightarrow A_x = K B_x$, $A_y = K B_y$, $= A_z = K B_z$

$\vec{A} = (A_x , A_y , A_z)$, $A = (K B_x , K B_y , K B_z)$, $\vec{A} = K (B_x , B_y , B_z)$, $\vec{A} = K B \rightarrow \vec{A}$ // $\vec{B}$

1) $\vec{A}$ // $\vec{B}$ in same direction if $K > 0$

2) $\vec{A}$ // $\vec{B}$in opposite direction if $K < 0$



if $\vec{A} = (2, -3, m)$ is parallel to the vector $\vec{B} = (1,n,8)$ find the values of m,n.

SOLUTION

$\vec{A}$ // $\vec{B} \rightarrow \frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}$

$\frac{2}{1} = -\frac{3}{n} = \frac{m}{8} \rightarrow -\frac{3}{n} = 2 \rightarrow n = -\frac{3}{2}$

$\frac{m}{8} = 2 \rightarrow m = 16$

EXAMPLE! (22)

if A= (2 , - 3) and $\vec{A}$ // $\vec{B}$ if $\|\vec{B}\| = 3\sqrt{13}$, find $\vec{B}$

SOLUTION

$\vec{A}$ = KB $\rightarrow \|\vec{A}\|$ = K $\|\vec{B}\| \rightarrow \sqrt{4 + 9} = 3\sqrt{13}$ K

K = $\frac{1}{3}$ $\vec{A} = \frac{1}{3} \rightarrow \vec{B}$ = (6, -9)

*** *The geometric meaning of the cross product of two vectors***

$\|\vec{A} \times \vec{B}\| = \|\vec{A}\| \, \|\vec{B}\| \sin\theta = \|\vec{B}\| \times$ L (where L = $\|\vec{A}\| \sin\theta$)

= Area of parallelogram where $\vec{A}$, $\vec{B}$ two adjacent sides in it = twice area of triangle in which $\vec{A}$, $\vec{B}$ two adjacent sides in it.

EXAMPLE! (23)

If $\vec{A}$ = (-3,1,2), $\vec{B}$ = (3,4,-1) Find the area of parallelogram in which $\vec{A}$, $\vec{B}$ are two adjacent sides in it.

SOLUTION

$$\vec{A} \times \vec{B} = (-3,1,2) \times (3,4,-1) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -3 & 1 & 2 \\ 3 & 4 & -1 \end{vmatrix}$$

= (- 1 – 8) $\hat{\imath}$ – (3 – 6) $\hat{\jmath}$ + (- 12 – 3) $\hat{k} \rightarrow \vec{A} \times \vec{B}$= - 9 $\hat{\imath}$ + 3 $\hat{\jmath}$ - 15 $\hat{k}$

$\|\vec{A} \times \vec{B}\| = \sqrt{9^2 + 3^2 + 15^2} = 3\sqrt{35}$ sq.u $\rightarrow$ Area of parallelogram = $3\sqrt{35}$ sq.u

The scalar triple product:

If $\vec{A}$, $\vec{B}$, $\vec{C}$ are three vectors

Then $\vec{A} \cdot \vec{B} \times \vec{C}$ is known as the scalar triple product

The expression $\vec{A} \cdot \vec{B} \times \vec{C}$ has no brackets where doing the scalar

Product first is meaningless

Let $\vec{A} = (A_x , A_y, A_z)$, $\vec{B} = (B_x,B_y,B_z)$, $\vec{C} = (C_x,C_y,C_z)$

$$\text{Then } \vec{A} \cdot \vec{B} \times \vec{C} = \vec{A} \cdot \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

= $\vec{A}$. [($B_y C_z – C_y B_z$) $\hat{\imath}$ – ($B_x C_z – C_x B_z$) $\hat{\jmath}$ + ($B_x C_y – C_x B_y$) $\hat{k}$]

= $A_x(B_y C_z – C_y B_z) – A_y (B_x C_z – C_x B_z) + A_z (B_x C_y – C_x B_y)$

$$\vec{A} \cdot \vec{B} \times \vec{C} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

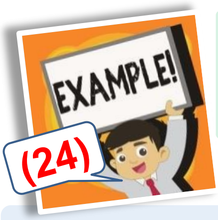
Properties of the scalar triple product

$\vec{A} \,.\, (\vec{B} \times \vec{C}) = \vec{B} \,.\, (\vec{C} \times \vec{A}) = \vec{C} \,.\, (\vec{A} \times \vec{B})$

The value of the scalar triple product doesn't change if the vectors in the same cyclic order

The geometric meaning of the scalar triple product

If $\vec{A}$, $\vec{B}$, $\vec{C}$ are three vectors and form three non parallel sides of parallelepiped then the volume of parallelepiped = the absolute value of the scalar triple product The volume of parallelepiped = $\left|\vec{A} \,.\, \vec{B} \times \vec{C}\right|$



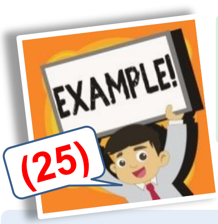
Find the volume of parallelepiped in which three adjacent sides are represented by the vectors A = (2,1,3) B = (-1,3,2) C = (1,1,-2)

SOLUTION

V = $\left|\vec{A} \,.\, (\vec{B} \times \vec{C})\right| \longrightarrow \vec{A} \,.\, \vec{B} \times \vec{C} = \begin{vmatrix} 2 & 1 & 3 \\ -1 & 3 & 2 \\ 1 & 1 & -2 \end{vmatrix}$

= 2 (-6 -2) - 1(2 -2) + 3(-1 -3) = -16 - 0 - 12 = -28

V = - 28 = 28 Cubic unit



Find the volume of the parallelepiped in which three non parallel edges are represented by the vectors $\vec{A}$ = (3,-4,1) $\vec{B}$ = (0,2,-3), $\vec{C}$ = (3,2,2)

SOLUTION

$\vec{A} \,.\, (\vec{B} \times \vec{C}) = \begin{vmatrix} 2 & 1 & 3 \\ -1 & 3 & 2 \\ 1 & 1 & -2 \end{vmatrix}$ = 3 (4 + 6) + 4 (0 + 9) + 1(0 - 6) = 30 + 36 – 6 = 60

Volume = $\left|\vec{A} \,.\, \vec{B} \times \vec{C}\right|$ 60 Cubic unit

EXERCISE 3

Answer the following:

S.B.2017

1) find $\vec{A} \bullet \vec{B}$ in each of the following :

(a) $\vec{A} = (5, 1, -2)$ and $\vec{B} = (4, -4, 3)$ [10]

(b) $\vec{A} = 3\,\hat{\imath} - 2\,\hat{\jmath} - \hat{k}$ and $\vec{B} = 6\,\hat{\imath} + 4\,\hat{\jmath} + 2\,\hat{k}$ [- 28]

(c) $\vec{A} = \hat{\imath}$ and $\vec{B} = 2\,\hat{\jmath} - \hat{k}$ [0]

S.B.2017

2) find the measure of the angle between the two vector in each of the following:

(a) (5, 1, - 2), (1, 1, - 1) [32.51°]

(b) (7, 2, -10), (2, 6, 4) [98.699°]

(c) (2, 1, 4), (1, -2, 0) [90°]

S.B.2017

3) ABCD is a rectangle in which AB = 6 cm , find:

(a) $\overrightarrow{AB} \bullet \overrightarrow{AC}$ [36]

(b) $\overrightarrow{AB} \bullet \overrightarrow{CD}$ [- 36]

(c) the component of $\overrightarrow{CD}$ in the direction of $\overrightarrow{BC}$ [0]

S.B.2017

4) in the opposite figure ;

A B C D A' B' C' D' is a cuboid

Find $\overrightarrow{BD} \bullet \overrightarrow{CA}$

S.B.2017

5) find the work done by the force $\vec{F} = (2, -3, 5)$ to move a body from the point (1, - 1, 0) to the point (2, 4, -2) [- 23 units of work]

S.B.2017

6) find the work done by the weight of a body of magnitude 40 Newton when projected vertically upwards for a distance of 10 meters above the surface of the ground. [- 400 joules]

Choose the correct answer in each of the following:

S.B.2017

7) if $\vec{A}$, $\vec{B}$ are two perpendicular unit vectors , then $(\vec{A} - 2\,\vec{B})* (3\,\vec{A} + 5\,\vec{B}) =$ …….

(a) - 8 (b) - 7 (c) 24 (d) 0

S.B.2017

8) if $\vec{A}$ and $\vec{B}$ are unit vectors , then $\vec{A} * \vec{B} \in$ …….

(a)] 0, 1 [(b)] – 1 , 1[(c) [- 1 , 1] (d) $\Re^{+}$

S.B.2017

9) the measure of the angle between the two vectors (2, -2, 2) and (1, 1, 4) is …….

(a) 57.02° (b) 35.26° (c) 134.37° (d) 0

S.B.2017

10) if θ is the measure of the angle included between the two vectors $\vec{A}$ = (-2, -6, 1) and $\vec{B}$ = (2, 6, -1) then θ = ……

(a) 30° (b) 60° (c) 120° (d) 180°

S.B.2017

11) if θ is the measure of the angle included between $\vec{A}$ = (2, 0, 2) and $\vec{B}$ = (0, 0, 4) then θ = ……

a) 30° (b) 45° (c) 60° (d) 90°

S.B.2017

12) if $\vec{A} = 2\,\hat{\imath} + 3\,\hat{\jmath} - \hat{k}$, $\vec{B} = 4\,\hat{\imath} - \hat{\jmath}$, then $\vec{A} * \vec{B}$ = ……

a) 5 (b) 4 (c) ± 3 (d) 2

S.B.2017

13) if $\vec{A}$ = (2, 3, -1) and $\vec{B} = 3\,\hat{\imath} + 4\,\hat{\jmath}$, then the component of $\vec{A}$ in the direction of $\vec{B}$ equals ……

a) 18 (b) $\frac{18}{5}$ (c) $-\frac{18}{5}$ (d) $\frac{18}{25}$

S.B.2017

14) if $\vec{A}$ = (1, -2, 1) and $\vec{B}$ = (- 2, 1, 2), then the vector component of $\vec{A}$ in the direction of $\vec{B}$ = ……

(a) $\left(\frac{4}{9}, \frac{-2}{9}, \frac{-4}{9}\right)$ (b) $\left(\frac{4}{9}, \frac{2}{9}, \frac{4}{9}\right)$ (c) $\left(\frac{-4}{9}, \frac{-2}{9}, \frac{-2}{9}\right)$ (d) $\left(\frac{4}{9}, \frac{2}{9}, \frac{-4}{9}\right)$

S.B.2017

15) find the work done by the force $\vec{F} = 3\,\hat{\imath} + 7\,\hat{k}$ to move a body from point A (1, 1, 2) to point B (7, 3, 5) equals …….

S.B.2017

16) if $\vec{A}$ = (-1, 4, 2) and $\vec{B}$ = (2, 2, 1), then find the component of $\vec{A}$ in the direction of $\vec{B}$ = ……

S.B.2017

17) if $\vec{A} = 2\,\hat{\imath} + 3\,\hat{\jmath} + m\,\hat{k}$, $\vec{B} = -6\,\hat{\imath} - 4\,\hat{\jmath} + 4\hat{k}$, and $\vec{A} \perp \vec{B}$, the m = ……

S.B.2017

18) in the opposite figure;

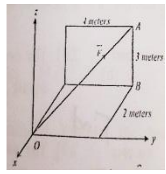
Find the value of k = ………

S.B.2017

19) $\vec{A}$, $\vec{B}$, $\vec{C}$ are three mutually perpendicular unit vectors, find $\left\| 2\vec{A} - \vec{B} + 3\,\vec{C} \right\|$ [$\sqrt{14}$]

MOE 2017

20) in the opposite figure;



The force $\vec{F}$ acts along $\overrightarrow{OA}$ where

F = 12 $\sqrt{29}$ Newton

(a) find the components of the force $\vec{F}$ [$-24\,\hat{\imath} + 48\,\hat{\jmath} + 36\,\hat{k}$]

(b) calculate the work done by the force $\vec{F}$ to move the body from the position O to the position B [240 N. m]

S.B.2017

21) prove that

(a) $\|\vec{A} + \vec{B}\|^2 + \|\vec{A} - \vec{B}\|^2 = 2\|\vec{A}\|^2 + 2\|\vec{B}\|^2$

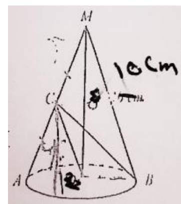
(b) $\|\vec{A} + \vec{B}\|^2 - \|\vec{A} - \vec{B}\|^2 = 4\,\vec{A} \bullet \vec{B}$

MOE 2017

22) if $\|\vec{A}\| = 3$, $\|\vec{B}\| = 4$, find $(\vec{A} + \vec{B}) \bullet (\vec{A} - \vec{B})$

S.B.2017

23) in the opposite figure;



The perimeter of the base of the right circular cone = 12 π cm, C is the mid-point of $\overline{AM}$,

then find $\overrightarrow{BC} \bullet \overrightarrow{CO}$ = ……

a) - 43 (b) - 40 (c) - 37 (d) – 33

S.B.2017

24) if $\vec{A}$ = (1, 6, 2), $\vec{B}$ = (k, 3, m) , $\vec{C}$ = (k, m, k + m) and $\vec{A}$ // $\vec{B}$, find $\|\vec{C}\|$ $\left[\frac{\sqrt{14}}{2}\right]$

S.B.2017

3) if $\vec{A}$ and $\vec{B}$ are two-unit vector in $\Re^3$ at what condition dose $\vec{A} \times \vec{B}$ represent a unit vector. Explain your answer.

S.B.2017

25) if $\vec{A} = \hat{\imath} + \hat{\jmath} - 2\,\hat{k}$ and $\vec{B} = 3\,\hat{\imath} - 2\,\hat{\jmath} + 5\,\hat{k}$, find

(a) $\vec{A} \times \vec{B}$ [$\hat{\imath} - 11\,\hat{\jmath} - 5\,\hat{k}$]

(b) $(2\,\vec{A}) \times (3\,\vec{B})$ [$6\,\hat{\imath} - 66\,\hat{\jmath} - 30\,\hat{k}$]

(c) $\vec{A} \times (\vec{A} - 2\vec{B})$ [$-2\,\hat{\imath} + 22\,\hat{\jmath} + 10\,\hat{k}$]

S.B.2017

26) find $\vec{A} \times \vec{B}$ in each of the following:

(a) $\vec{A}$ = (-2, 3, 1) and $\vec{B}$ = (1, 3, - 4) [$-15\,\hat{\imath} - 7\,\hat{\jmath} - 9\,\hat{k}$]

(b) $\vec{A} = -\hat{\imath} - 2\,\hat{\jmath} + 5\,\hat{k}$ and $\vec{B} = 3\,\hat{\imath} - 5\,\hat{k}$ [$10\,\hat{\imath} - 5\,\hat{\jmath} - 3\,\hat{k}$]

(c) $\|\vec{A}\| = 6$, $\|\vec{B}\| = 8$ and the angle between them is 60° [$24\sqrt{3}\,\vec{C}$]