Example (14)

A wire of length 64 cm is cut into two portions , the first is bent to form a square , and the second to form a circle . Find the length of each portion if the sum of the surface areas of the square and the circle is minimum .

Calculus $0.27t - 3.$ $-4t + 24t - 104$ (x^2) *
* $\therefore x = \frac{1}{2}(32 - \pi r) - (-11)$ *

And Area* $(A) = x^2 + \pi r^2 = \frac{1}{4}(32 - \pi r)^2 + \pi r^2$ *
* $(Ar) = 256 - 16\pi r + \frac{1}{4}\pi^2 r^2 + \pi r^2 \Rightarrow \therefore A'(r) = -16\pi + \frac{1}{2}\pi^2 r + 2\pi r$ *

When A' = 0 \Rightarrow \therefore -16\pi + \frac{1}{2 r In this problem, we have to make a relation between Area of the two portions and their perimeters* Figures
 $\frac{(\text{even Area of the two portions an})}{(\text{inometric area of the two portions})}$
 $\frac{(\text{odd area of the two portions})}{(\text{odd area of the two portions})}$ In this problem, we have to make a relation between A
 \therefore The length of the wire = the length of the two por
 \therefore Let the perimeter of the square be 4x and the per

of the circle be $2\pi r \implies \therefore 4x + 2\pi r = 64 \ (\div 2)$
 verimeter of the square be 4x and the perimeter

rcle be $2\pi r \implies$ \therefore $4x + 2\pi r = 64 \quad (\div 2)$
 $\overline{32 - \pi r}$ $\overline{) - -(-1)}$
 $(A) = x^2 + \pi r^2 = \frac{1}{4}(32 - \pi r)^2 + \pi r^2$ *The length of the circle is minimum*
 The length of the wire = the length of the two portions = 64 cm
 *Let the perimeter of the sauare be 4x and the perimeter <u>Answer</u>

<i>i* this problem, we have to make a relation between Area of t
 The length of the wire = the length of the two portions =
 Let the perimeter of the square be 4x and the perimeter
 of the circle be $2\pi r$ *1 x* = *let* the perimeter of the *s*
 z the circle be $2\pi r$ \Rightarrow
 $x = \frac{1}{2}(32 - \pi r) - (-1)^2$ *1 of the circle be* $2\pi r \Rightarrow$ \therefore $4x + 2\pi r = 64 \quad (\div 2)$
 \therefore $x = \frac{1}{2}(32 - \pi r) - (-1)$
 And Area $(A) = x^2 + \pi r^2 = \frac{1}{4}(32 - \pi r)^2 + \pi r^2$ have to make a relation between Area of
wire = the length of the two portions
of the square be 4x and the perimete
 $\pi r \implies \therefore 4x + 2\pi r = 64 \ (\div 2)$ $\frac{\text{Answer}}{\text{64.444}}$
and the tength of dethermining the standard of the two portions as
= the length of the two portions = 64 cm
 $\frac{\text{64.44}}{\text{64.44}}$ cm
= square be 4x and the perimeter \mathcal{L}_{\bullet} Fo make a relation between Area of the two p
= the length of the two portions = 64 cm
e square be 4x and the perimeter
 \Rightarrow \therefore $4x + 2\pi r = 64$ (\div 2) The length of the wire = the length of th
Let the perimeter of the square be 4x an
of the circle be $2\pi r \Rightarrow$ \therefore $4x + 2\pi r =$
 \therefore $x = \frac{1}{2}(32 - \pi r) - (-1)$ $x = \frac{1}{2}(32 - \pi r) - (-1)$

Area $(A) = x^2 + \pi r^2 = \frac{1}{4}(32 - \pi r)^2 + \pi r^2$
 $(r) = 256 - 16\pi r + \frac{1}{4}\pi^2 r^2 + \pi r^2 \implies \therefore A'(r) = -16\pi + \frac{1}{2}\pi^2 r + 2\pi r$ πr^2

A'(r) = -16 π + $\frac{1}{2}\pi^2 r$ + $2\pi r$

((x 2) And ($\div \pi$)) $A'(r) = -16\pi + \frac{1}{2}\pi^2 r + 2\pi r$

((x 2) And (÷ π))

(2) en $A' = 0 \implies \therefore -16\pi + \frac{1}{2}$
 $-32 + \pi r + 4r = 0 \implies$
 $(r) = \frac{1}{2}\pi^2 + 2\pi > 0 \implies \therefore$ $=\frac{1}{4}(32 - \pi r)^2 + \pi r^2$
 $\frac{1}{2}r^2 + \pi r^2 \implies \therefore A'(r) = -16\pi + \frac{1}{2}\pi^2$ *2 2* \therefore $x = \frac{1}{2}(32 - \pi r) - (-1)$
 nd Area (A) = $x^2 + \pi r^2 = \frac{1}{4}(32 - \pi r)^2 + \pi r^2$

A(r) = 256 − 16 $\pi r + \frac{1}{4}\pi^2 r^2 + \pi r^2 \implies$ ∴ A'(r) = -16 $\pi + \frac{1}{2}\pi^2 r + 2\pi r$ $\frac{d}{dr}$
 $\frac{d}{dt}r^2 = \frac{1}{4}(32 - \pi r)^2 + \pi r^2$
 $\frac{d}{dt}\pi^2 r^2 + \pi r^2 \implies \therefore A'(r) = -16\pi + \frac{1}{2}$ And Area $(A) = x^2 + \pi r^2 = \frac{1}{4}(32 - \pi r)^2 + \pi r^2$
 $\therefore A(r) = 256 - 16\pi r + \frac{1}{4}\pi^2 r^2 + \pi r^2 \implies \therefore A'(r) = -16\pi + \frac{1}{2}$

When $A' = 0 \implies \therefore -16\pi + \frac{1}{2}\pi^2 r + 2\pi r = 0$ ((x 2) And ($\div \pi$ $32 + \pi r + 4r = 0 \implies r^2 - 16\pi + \frac{1}{2}\pi^2 r + 2\pi r = 0$ ((x
 $32 + \pi r + 4r = 0 \implies r = \frac{32}{\pi + 4} - (-2)$ *2* $\frac{1}{2}$ *A''* $\frac{1}{2}$ *A''* $\frac{1}{2}$ *A''* $\frac{1}{2}$ *A''* $\frac{1}{2}$ *A''* $\frac{1}{\pi + 4}$ *A''* $\left(r\right) = \frac{1}{2}$ *π*² + 2*π* > 0 \Rightarrow *∴* area of the two portions are minimum when $r = \frac{32}{\pi + 4}$ $\pi r + 4r = 0 \Rightarrow \boxed{r = \frac{32}{\pi + 4} - (-2)}$
 $\frac{1}{2} \pi^2 + 2\pi > 0 \Rightarrow \therefore$ area of the two portions are minimum when $r = \frac{32}{\pi + 4}$ *So by substitut* And Area $(A) = x^2 + \pi r^2 = \frac{1}{4}(32 - \pi r)^2 + \pi r^2$
 $\therefore A(r) = 256 - 16\pi r + \frac{1}{4}\pi^2 r^2 + \pi r^2 \implies \therefore A'(r) = -16\pi + \frac{1}{2}\pi^2 r + 2\pi r$ π $r+4r=0 \Rightarrow r+\frac{3}{2}r^2$
 $r+4r=0 \Rightarrow r=\frac{3}{\pi}$
 $r^2+2\pi>0 \Rightarrow \therefore \text{ are }$ $A = 0 \implies . -10\pi + \frac{1}{2}\pi r + 2\pi r = 0 \quad (\times 2)$ And $(\pi \pi)$
+ $\pi r + 4r = 0 \implies \boxed{r = \frac{32}{\pi + 4} - (-2)}$
= $\frac{1}{2}\pi^2 + 2\pi > 0 \implies .$ area of the two portions are minimum when $r = \frac{32}{\pi + 4}$ $a(A) = x^2 + \pi r^2 = \frac{1}{4}(32 - \pi r)^2 + \pi r^2$
 $256 - 16\pi r + \frac{1}{4}\pi^2 r^2 + \pi r^2 \implies \therefore A'(r) = -16\pi + \frac{1}{2}\pi^2 r + 2\pi r$
 $= 0 \implies \therefore -16\pi + \frac{1}{2}\pi^2 r + 2\pi r = 0 ((\times 2) \text{ And } (\div \pi))$ ∴ $A(r) = 256 - 16\pi r + \frac{1}{4}\pi^2 r^2 + \pi r^2 \implies$ ∴ $A'(r) = -16\pi + \frac{1}{2}$

When $A' = 0 \implies$ ∴ $-16\pi + \frac{1}{2}\pi^2 r + 2\pi r = 0$ ((× 2) And (÷ π

∴ $-32 + \pi r + 4r = 0 \implies r = \frac{32}{\pi + 4} - (2)$ + 0 ⇒ $\boxed{r = \frac{32}{\pi + 4} - -(2)}$

> 0 ⇒ ∴ area of the two ports

(2) in (1) : $x = \frac{1}{2} \left(32 - \pi \left(\frac{32}{\pi + 4} \right) \right)$ $\frac{1 - 7x - 7x}{\pi + 4}$ *2 1* $r = 0 \Rightarrow$ $r = \frac{32}{\pi + 4} - (-2)$
 $-2\pi > 0 \Rightarrow$ \therefore area of the two portions are minimum when $r = \frac{32}{\pi + 4}$
 ing (2) *in* (1) \therefore $x = \frac{1}{2} \left(32 - \pi \left(\frac{32}{\pi + 4} \right) \right) = 16 - \frac{16\pi}{\pi + 4} = \frac{16\pi + 64 - 16\pi}{\pi + 4}$ *² ⁴ 4 4 4 256 The length of the first portion P 4x* So by substituting (2) in (1) : $x = \frac{1}{2} \left(32 - \pi \left(\frac{32}{\pi + 4} \right) \right) = 16 - \frac{16}{\pi}$
 \therefore The length of the first portion $P_1 = 4x = \frac{256}{\pi + 4}$

And the length of the second portion $P_2 = 2\pi r = 2\pi \times \frac{32}{\pi + 4} = \frac$ $\pi + 4$
 $\frac{64\pi}{\pi + 4}$ mum when $r = \frac{32}{\pi + 4}$
 $\frac{\pi}{4} = \frac{16\pi + 64 - 16\pi}{\pi + 4} = \frac{64}{\pi + 4}$ π portions are minimum when $r = \frac{32}{\pi + 4}$
 $(\frac{32}{\pi + 4})$ = $16 - \frac{16\pi}{\pi + 4} = \frac{16\pi + 64 - 16\pi}{\pi + 4} = \frac{64}{\pi + 4}$ π $\pi\left(\frac{\pi}{\pi+4}\right)$ = 10 - $\frac{\pi}{\pi+4}$
256
 $\pi+4$
 $\pi r = 2\pi \times \frac{32}{\pi+4} = \frac{64\pi}{\pi+4}$ $\pi + 4$ $\pi + 4$
 $\frac{32}{\pi + 4} = \frac{64\pi}{\pi + 4}$ $\frac{1}{\pi}$ = ---(2)

of the two portions are minimum when $r = \frac{32}{\pi + 4}$
 $\left(32 - \pi \left(\frac{32}{\pi + 4}\right)\right) = 16 - \frac{16\pi}{\pi + 4} = \frac{16\pi + 64 - 16\pi}{\pi + 4} = \frac{64}{\pi + 4}$ $\frac{1}{\pi + 4}$ = - - (2)

ea of the two portions are minimum when $r = \frac{32}{\pi + 4}$
 $= \frac{1}{2} \left(32 - \pi \left(\frac{32}{\pi + 4} \right) \right) = 16 - \frac{16\pi}{\pi + 4} = \frac{16\pi + 64 - 16\pi}{\pi + 4} = \frac{64}{\pi + 4}$
 $= 256$ So by substituting (2) in (1) : $x = \frac{1}{2} \left(32 - \pi \left(\frac{32}{\pi + 4} \right) \right) = 16$
 \therefore The length of the first portion $P_1 = 4x = \frac{256}{\pi + 4}$ $2 - \pi \left(\frac{\pi}{\pi} + 4 \right)$ = 10 - $\frac{\pi}{\pi} + 4$
= $\frac{256}{\pi + 4}$
= $2\pi r = 2\pi \times \frac{32}{\pi + 4} = \frac{64\pi}{\pi + 4}$ $\pi + 4$ $\pi + 4$
 $\frac{32}{+4} = \frac{64\pi}{\pi + 4}$ *Answer*

Example (15)

Example (15)
A piece of land in the form of a trapezium ABCD in which \overline{AB} *//* \overline{CD} *,* $\overline{BC} \perp \overline{AB}$ *,* $AB = 15$ *m
* $BC = 20$ *m,* $CD = 5$ *m. It is reauired to build a house on a portion of this land in the shape of Bxample* (15)
BC = 20 m, CD = 5 m. It is required to build a house on a portion of this land in the shape of a
BC = 20 m, CD = 5 m. It is required to build a house on a portion of this land in the shape of a
rectang rectangle so that the point O is taken on AD and OH is drawn $\perp AB$, and $ON \perp BC$. $\pm \overline{AB}$, $AB = 15$ m
d in the shape of a *he form of a trapezium ABCD in which* \overline{AB} *//* \overline{CD} *,* $\overline{BC} \perp \overline{AB}$ *,* $AB = 15$ *m***
** \overline{BD} *m. It is required to build a house on a portion of this land in the shape of a***
** *he point O is taken on* \overline{AD} *and Find the maximum surface area of the rectangle OHBN .*

Answer

In this problem, we have to make a relation between area of the rectangle and the sides of trapezuim

Calculus of $P(X) = \frac{1}{2}$ $\frac{1}{2}$ *\frac{* $\begin{bmatrix} C & 5 & D \\ 20 & x & 3 \end{bmatrix}$
 D $\begin{bmatrix} x & y & 5 \\ x & 5 & y-5 \\ y & 5 & F & E \\ \hline \end{bmatrix}$ *O H N x* $20 - x$ *y* $15 - y$ *F* $5 \mid y-5$ mgth of the rectangle be x and its width be y

can't make the relation except if we join \overline{DF}
 $\triangle AHO \sim \triangle AFD \Rightarrow \therefore \frac{AH}{AF} = \frac{HO}{FD}$
 $\therefore \frac{x}{20} \Rightarrow 15 - y = \frac{x}{2} \Rightarrow \frac{[x + 30 - 2y - - -]}{[x + 30 - 2y - -]}$

(area of rectangle) = $xy = (3$ *Let the length of the rectangle be x and its width be y Here we can't make the relation except if we join DF Let the length of the rectangle be x and its w*
Here we can't make the relation except if we
To make $\triangle AHO \sim \triangle AFD \Rightarrow \therefore \frac{AH}{AF} = \frac{HO}{FD}$ *A AH* $\frac{A}{AF} = \frac{HO}{FD}$ *Here we can't make the relation with make* $\triangle AHO \sim \triangle AFD$ *
* $\frac{15 - y}{10} = \frac{x}{20} \Rightarrow 15 - y = \frac{x}{20}$ ake the relation except if we join \overline{DF}
 $0 \sim \Delta AFD \Rightarrow \therefore \frac{AH}{AF} = \frac{HO}{FD}$
 $15 - y = \frac{x}{2} \Rightarrow \boxed{\therefore x = 30 - 2y - (-1)^2}$ *b* make $\triangle AHO \sim \triangle AFD$
 $\frac{5-y}{10} = \frac{x}{20} \Rightarrow 15 - y = \frac{x}{2}$
 nd : A (area of rectangle) *And A area of rectangle x y 30* mgth of the rectangle be x and its width be y
can't make the relation except if we join \overline{DF}
 $\triangle AHO \sim \triangle AFD \Rightarrow \therefore \frac{AH}{AF} = \frac{HO}{FD}$ − ve can't make the relation except if we join \overline{DF}
 $= \frac{x}{20} \Rightarrow 15 - y = \frac{x}{2} \Rightarrow \overline{-. \cdot x = 30 - 2y - (-1)}$
 $A(\text{area of rectangle}) = xy = (30 - 2y)y - (-2)$
 $= 30y - 2y^2 \Rightarrow ... A' = 30 - 4y$ (3) $\begin{array}{c|c|c|c}\n\hline\n & x & & \\
\hline\n & 5 & y-5 & 15 \\
\hline\n & & 5 & F & H & \n\hline\n & 15 & & \n\hline\n & 15 & & \n\hline\n\end{array}$ $-4y=0$ \Rightarrow $y = \frac{30}{4} = \frac{15}{2} - (-3)$
so the area of the rectangle is maximum at $y = \frac{15}{2}$
(2) \therefore $A_{Maximum} = (30-2y)y = 15 \times \frac{15}{2} = \frac{225}{2} = 1$ \therefore $A = 30y - 2y^2 \implies$ \therefore $A' = 30 - 4y$ $\frac{2}{2}$
 $\frac{-2y}{-(-2)}$
 $\frac{2}{2}y$
 $\frac{2}{y}$
 $\frac{2}{-(-2)}$ $\frac{5-y}{10} = \frac{x}{20} \implies 15 - y = \frac{x}{2} \implies \therefore x =$
 A \therefore *A* (*area of rectangle*) = *x y* = (3)
 A = 30 *y* - 2 *y*² \implies \therefore *A'* = 30 - 4 *y 30* 2 *15 3* \rightarrow <u>2</u> \rightarrow <u>[... *x* 5*3* 2*3* (1)]
 And: *A*(*area of rectangle*) = *x* $y = (30 - 2y)y - --(2)$
 i. *A* = 30*y* - 2*y*² \Rightarrow *i*. *A'* = 30 - 4*y*
 When A' = 0 \Rightarrow *i*. 30 - 4*y* = 0 \Rightarrow $\boxed{y = \frac$ *When* $A' = 0 \implies$ \therefore $30 - 4y = 0 \implies y = \frac{30}{4} = \frac{15}{2} - (-3)$ *15 And* \therefore *A'* = *-4* $\lt 0 \Rightarrow$ \therefore *A'* = *-4* $\lt 0 \Rightarrow$ *So the area of the rectangle is maximum at y* = $\frac{15}{2}$ *When* $A' = 0 \implies$ $\therefore 30 - 4y = 0 \implies y = \frac{90}{4} = \frac{15}{2} - (-3)$
 And $\therefore A'' = -4 < 0 \implies$ *so the area of the rectangle is maximum at* $y = \frac{15}{2}$
 Then by substituting in (2) $\therefore A_{Maximum} = (30 - 2y)y = 15 \times \frac{15}{2} = \frac{225}{2} = 112.5$ *n* at $y = \frac{15}{2} = \frac{22}{2}$ $\frac{10}{F D}$ 20
 $\left|\begin{array}{c} x \\ x \\ y \\ -2y \end{array}\right|$ $\left|\begin{array}{c} y \\ y \\ y \\ z \end{array}\right|$ $\frac{15 - y}{10} = \frac{x}{20} \implies 15 - y = \frac{x}{2} \implies \boxed{\therefore x = 30 - 2y - -1}$
And : A(area of rectangle) = $xy = (30 - 2y)y - -1$
 $\therefore A = 30y - 2y^2 \implies \therefore A' = 30 - 4y$ $(area of rectangle) = x y = (30 - 2y)y - --(2)$
 $y - 2y^2$ ⇒ ∴ $A' = 30 - 4y$
 $= 0$ ⇒ ∴ $30 - 4y = 0$ ⇒ $y = \frac{30}{4} = \frac{15}{2} - -(-3)$ $y = \sqrt{y - \frac{y^2}{4} - \frac{z^2}{2} - (-9)^2}$

urea of the rectangle is maximum at $y = \frac{15}{2}$
 $\therefore A_{Maximum} = (30 - 2y) y = 15 \times \frac{15}{2} = \frac{225}{2} = 112.5$ cm².

2 Maximum

Example (16)

and gluing the cut edges of the remaining piece together (see the figure) *A cone is made from a circular sheet of raduis R by cutting out a sector What is the maximum volume attainable for this cone .*

z z z z z z z z z <i>z 2 and $\left[\because r^2 = R^2 - h^2\right] \implies \therefore V = \frac{1}{2}\pi h(R^2 - h^2) = \frac{\pi}{2}R^2h - \frac{\pi}{2}h^3$ $\frac{\pi h\left(R^2\right)^2}{\left(\pi h\right)^2}$ *1* gluing the cut edges of the remaining piece together (see the figure)
 at is the maximum volume attainable for this cone.
 $V_{cone} = \frac{1}{3}\pi r^2 h$ and $\therefore r^2 = R^2 - h^2 \implies \therefore V = \frac{1}{3}\pi h (R^2 - h^2) = \frac{\pi}{3} R^2 h - \frac{\pi}{3} h$ *s a anximum volume attainable for this cone* .

<u>*Answer*</u>
 3 $\pi r^2 h$ and $\therefore r^2 = R^2 - h^2$ $\Rightarrow \therefore V = \frac{1}{3} \pi h (R^2 - h^2) = \frac{\pi}{3} R^2 h - \frac{\pi}{3}$ $V_{cone} = \frac{1}{3}\pi r^2 h$ and $\boxed{\therefore r^2 = R^2 - h^2} \Rightarrow \therefore V = \frac{1}{3}\pi h (R^2)$
Differentiate w.r. to h (as R is given) $\Rightarrow \therefore V' = \frac{\pi}{3} R^2 - \pi h$ ng the cut edges of the remaining piece together (see the figure)

the maximum volume attainable for this cone.
 $\frac{\text{Answer}}{3} \pi r^2 h$ and $\boxed{\therefore r^2 = R^2 - h^2} \Rightarrow \therefore V = \frac{1}{3} \pi h (R^2 - h^2) = \frac{\pi}{3} R^2 h - \frac{\pi}{3} h^3$ $\frac{Answer}{3}$

∴ $V_{cone} = \frac{1}{3}\pi r^2 h$ and $\frac{r^2 = R^2 - h^2}{r^2} \Rightarrow$ ∴ $V = \frac{1}{3}\pi h(R^2 - h^2) = \frac{\pi}{3} F$

∴ Differentiate w.r. to h (as R is given) ⇒ ∴ $V' = \frac{\pi}{3} R^2 - \pi h^2$ *Answer*

3 π π

$$
V_{cone} = \frac{1}{3}\pi r^2 h \quad \text{and} \quad \boxed{\therefore r^2 = R^2 - h^2} \Rightarrow \therefore V = \frac{1}{3}\pi h (R^2 - h^2) = \frac{\pi}{3} h^2
$$
\n
$$
\therefore \text{ Differentiate w.r. to h (as R is given)} \Rightarrow \therefore V' = \frac{\pi}{3} R^2 - \pi h^2
$$
\n
$$
\text{When } V' = 0 \Rightarrow \therefore \frac{\pi}{3} R^2 - \pi h^2 = 0 \Rightarrow h^2 = \frac{R^2}{3} \Rightarrow \boxed{\therefore h = \frac{R}{\sqrt{3}}}
$$
\n
$$
V'' = -2\pi h < 0 \Rightarrow \therefore \text{ the maximum volume occurs at } h = \frac{R}{\sqrt{3}} \text{ and its value}
$$

R When $V' = 0 \implies \therefore \frac{\pi}{3} R^2 - \pi h^2 = 0 \implies h^2 = \frac{R^2}{3} \implies \left[\therefore h = \frac{R}{\sqrt{3}} \right]$
 $V'' = -2\pi h < 0 \implies \therefore$ the maximum volume occurs at $h = \frac{R}{\sqrt{3}}$ and its value is *maximum volume occurs at* $h = \frac{R}{\sqrt{3}}$ *and its value is:
* $\frac{3}{\pi} = \pi R^3 = \pi R^3 = 2\pi R^3$ *2 R R* $\left(\frac{R}{3}\right)$ *R* $\left(\frac{R}{\sqrt{3}}\right)$ *R* $\left(\frac{R}{\sqrt{3}}\right)^3$ *R* $\left(\frac{R}{\sqrt{3}}\$ $\frac{\pi}{3} R^2 \left(\frac{R}{\sqrt{3}} \right) - \frac{\pi}{3} \left(\frac{R}{\sqrt{3}} \right)^3 = \frac{\pi R^3}{3\sqrt{3}} - \frac{\pi R^3}{9\sqrt{3}} = \frac{2\pi R}{9\sqrt{3}}$ $n \text{ } v = 0 \implies ... \frac{1}{3} R - \pi n = 0 \implies n = \frac{1}{3} \implies ... n = \frac{1}{\sqrt{3}}$
= $-2\pi h < 0 \implies ...$ the maximum volume occurs at $h = \frac{R}{\sqrt{3}}$ a
 $\frac{\pi}{3} R^2 \left(\frac{R}{\sqrt{3}} \right) - \frac{\pi}{3} \left(\frac{R}{\sqrt{3}} \right)^3 = \frac{\pi R^3}{3\sqrt{3}} - \frac{\pi R^3}{9\sqrt{3}} = \frac{2\pi R^3}{9\sqrt{3}}$ S
 $S' = -2\pi h < 0 \implies$: the maximum volume occurs at $h = \frac{1}{\sqrt{2}}$
 $= \frac{\pi}{3} R^2 \left(\frac{R}{\sqrt{3}} \right) - \frac{\pi}{3} \left(\frac{R}{\sqrt{3}} \right)^3 = \frac{\pi R^3}{3\sqrt{3}} - \frac{\pi R^3}{9\sqrt{3}} = \frac{2\pi R^3}{9\sqrt{3}}$

Example (17)

A man in a boat 14 metres from the nearest point A on the straight shore wishes to go to a point B 60 metres down the shore from A. He can land any where on the shore between A and B and walks the rest of the way . He can row with velocity 30 metres / minute and walks with velocity 40 metres / min . Where should he lands in order to reach B in the least time .

Answer

The word nearest here will make me draw the boat perpendicular to A

Also the word least time means that we have to find the time in each part of the problem So let the boat reaches the shore at C The b oat will reach point B when it passes have to

m
 $B \frac{60-x}{20m/m}$

asses

(t) minutes

covers *2 2 rough MC then BC will make*
 here MC = $\sqrt{x^2 + 196}$ which
 ith velocity 30 m/sec.
 Its time $T_1 = \frac{d}{v} = \frac{\sqrt{x^2 + 196}}{30}$ *through MC then BC will make a time* (t) *minutes So let the boat reaches the shore at C*
 \therefore *The boat will reach point B when it passes*
 through MC then BC will make a time (*t*) *minu*
 Where MC = $\sqrt{x^2 + 196}$ *which the boat covers*
 with velocity 30 m/sec with velocity 30 m / sec . $\frac{2}{v} + 196$ w.
 v m / sec.
 $\frac{d}{v} = \frac{\sqrt{x^2 + 30}}{30}$

hich the ma *with velocity* 30 *m* / sec.
 \therefore *Its time* $T_1 = \frac{d}{v} = \frac{\sqrt{x^2 + 19}}{30}$
 $CB = 60 - x$ *which the man*
 $d = 60 - x$ Where $MC = \sqrt{x^2 + 196}$ which the *d*
with velocity 30 m/sec.
 \therefore Its time $T_1 = \frac{d}{v} = \frac{\sqrt{x^2 + 196}}{30}$ $\frac{dy}{dx}$ velocity 30 m/sec.

Its time $T_1 = \frac{d}{v} = \frac{\sqrt{x^2 + 1}}{30}$

= 60 - x which the m ers walking by velocity 40 m/min.

otal time will cover by the boat to reach
 $(60-x)---(1)$ Its time $T_1 = \frac{d}{v} = \frac{\sqrt{x^2 + 196}}{30}$
 $B = 60 - x$ which the man covers walking by velocity 40 m/min.
 $T_2 = \frac{d}{v} = \frac{60 - x}{40}$ \Rightarrow \therefore the total time will cover by the boat to reach B is: $T = T_1 + T_2$ *2* $CB = 60 - x$ which the man covers walking by velocity 40 m/min. *v me* $T_1 =$
 v – x w
 w $\frac{d}{v} = \frac{60 - 40}{40}$ *CB* = 60 - *x* which the man covers walking by v
 $T_2 = \frac{d}{v} = \frac{60 - x}{40}$ \Rightarrow \therefore the total time will co

Where $t = \frac{1}{30}\sqrt{x^2 + 196} + \frac{1}{40}(60 - x) - (-1)^2$ $T_2 = \frac{V}{v} = \frac{40}{40}$

Where $t = \frac{1}{30} \sqrt{x^2}$
 $\frac{dt}{dx} = \frac{1}{30} \times \frac{2}{30}$ Where $\frac{dt}{dx} = \frac{1}{30}$ ∴ Its time $T_1 = \frac{d}{v} = \frac{\sqrt{x^2 + 196}}{30}$

CB = 60 - x which the man covers walking by velocity 40 m/min.

∴ $T_2 = \frac{d}{v} = \frac{60 - x}{40}$ \Rightarrow ∴ the total time will cover by the boat to reach B is: $T = T_1 + T_2$ x which the man covers walking by velocity 40 m.
= $\frac{60-x}{40}$ \Rightarrow : the total time will cover by the bod
= $\frac{1}{30}\sqrt{x^2+196}$ + $\frac{1}{40}(60-x)$ - - - (1) ere $\frac{1}{t} = \frac{1}{30} \sqrt{x^2 + 196}$
= $\frac{1}{30} \times \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2 + 196}}$ *x* $\frac{1}{20}$ \Rightarrow \therefore the total time will cover by the boat to reach B is:
 $\frac{1}{20}\sqrt{x^2+196} + \frac{1}{40}(60-x) - (-1)$
 $\frac{2x}{2\sqrt{x^2+196}} - \frac{1}{40} \Rightarrow \therefore$ when $\frac{dt}{dx} = 0 \Rightarrow \frac{x}{30\sqrt{x^2+196}} - \frac{1}{40} = 0$ $(Square both)$ *2* $2\sqrt{x^2 + 196}$ 40
 $\frac{x}{x^2 + 196} = \frac{1}{40} \implies \therefore \frac{x}{\sqrt{x^2 + 196}} = \frac{3}{4}$ (Square both) $\implies \therefore \frac{x}{x^2}$ $rac{x}{\sqrt{x^2 + 196}} = \frac{1}{40} \Rightarrow$ $\therefore \frac{x}{\sqrt{x^2 + 196}} = \frac{3}{4}$ (Sque
 $x^2 = 9x^2 + 1764 \Rightarrow$ $\therefore 7x^2 = 1764 \Rightarrow$ $\therefore x^2 = 1764$ $\frac{x^2 + 196}{x^2 + 196} + \frac{1}{40}(60 - x) - (-1)$
 $\frac{x}{x^2 + 196} - \frac{1}{40} \implies \therefore \text{ when } \frac{dt}{dx} = 0 \implies \frac{x}{30\sqrt{x^2 + 196}} - \frac{1}{40}$ $\frac{2x}{\sqrt[3]{x^2 + 196}} - \frac{1}{40} \implies \therefore \text{ when } \frac{dt}{dx} = 0 \implies \frac{x}{30\sqrt{x^2 + 196}} - \frac{1}{40} = 0$
 $\frac{x}{\sqrt[3]{x^2 + 196}} = \frac{1}{40} \implies \therefore \frac{x}{\sqrt{x^2 + 196}} = \frac{3}{4}$ (Square both) $\implies \therefore \frac{x^2}{x^2 + 196} = \frac{9}{16}$ $\frac{40}{5}$ \Rightarrow \therefore when $\frac{dt}{dx} = 0$ \Rightarrow $\frac{30\sqrt{x^3}}{\sqrt{x^2 + 196}} = \frac{3}{4}$ (Square both) $\frac{1}{30} \times \frac{\cancel{x}}{\cancel{x} \sqrt{x^2 + 196}} - \frac{1}{40} \implies \therefore \text{ when } \frac{dt}{dx} = 0 \implies \frac{x}{30\sqrt{x^2 + 196}} - \frac{1}{40} = 0$
 $\frac{x}{30\sqrt{x^2 + 196}} = \frac{1}{40} \implies \therefore \frac{x}{\sqrt{x^2 + 196}} = \frac{3}{4} \text{ (Square both)} \implies \therefore \frac{x^2}{x^2 + 196} = \frac{9}{16}$ 30 $\sqrt{x^2 + 196}$ 40 dx 30 $\sqrt{x^2 + 196}$ 40
 $\frac{x}{30\sqrt{x^2 + 196}} = \frac{1}{40} \implies \therefore \frac{x}{\sqrt{x^2 + 196}} = \frac{3}{4}$ (Square both) $\implies \therefore \frac{x^2}{x^2 + 196}$
 $16x^2 = 9x^2 + 1764$ $\implies \therefore 7x^2 = 1764 \implies \therefore x^2 = 252 \implies \therefore x = 6\sqrt{7}$ m $\frac{1}{40}(60-x)$ - - - - (1)
 $-\frac{1}{40}$ \Rightarrow : when $\frac{dt}{dx} = 0$ \Rightarrow $\frac{x}{30\sqrt{x^2 + 196}} - \frac{1}{40} = 0$ $\frac{1}{x+196} + \frac{1}{40}(60-x) - (-1)$
 $\frac{x}{x+196} - \frac{1}{40} \implies \therefore \text{ when } \frac{dt}{dx} = 0 \implies \frac{x}{30\sqrt{x^2+196}} - \frac{1}{40} = 0$ $rac{dt}{dx} = \frac{1}{30} \times \frac{2x}{2\sqrt{x^2 + 196}} - \frac{1}{40} \implies \therefore \text{ when } \frac{dt}{dx} = 0 \implies \frac{x}{30\sqrt{x^2 + 196}} - \frac{1}{40} = 0$

∴ $\frac{x}{30\sqrt{x^2 + 196}} = \frac{1}{40} \implies \therefore \frac{x}{\sqrt{x^2 + 196}} = \frac{3}{4}$ (Square both) $\implies \therefore \frac{x^2}{x^2 + 196} = \frac{9}{16}$ $\frac{\sqrt{2x}}{\sqrt[3]{x^2 + 196}} - \frac{1}{40} \Rightarrow$ \therefore when $\frac{dt}{dx} = 0 \Rightarrow \frac{x}{30\sqrt{x^2 + 196}} - \frac{1}{40} = 0$
 $\frac{1}{\sqrt[3]{x^2 + 196}} = \frac{1}{40} \Rightarrow$ $\therefore \frac{x}{\sqrt{x^2 + 196}} = \frac{3}{4}$ (Square both) $\Rightarrow \frac{x}{x^2 + 196} = \frac{3}{4}$ dx 30 $\sqrt{x^2 + 196}$ 40 \rightarrow \cdots men dx \rightarrow \rightarrow 30 $\sqrt{x^2 + 196}$ 40 \rightarrow
 $\therefore \frac{x}{30\sqrt{x^2 + 196}} = \frac{1}{40} \Rightarrow \therefore \frac{x}{\sqrt{x^2 + 196}} = \frac{3}{4}$ (Square both) \Rightarrow $\therefore \frac{x^2}{x^2 + 196} = \frac{9}{16}$
 $\therefore 16x^2 = 9x^2 + 1764 \Rightarrow \therefore 7x^2 = 176$ $B \leq$ A $\begin{array}{c}\n x \\
 \hline\n x \\
 \hline\n 30m \angle min.\n \end{array}$ *30m / min. 40m / min. ^C M* $60 - x$ *60* $f'(0) = -ve$ $\frac{|x = 6\sqrt{7}|}{|}$ $f'(20) = +ve$ *Then the least time needed by the boat to reach point B is when the boat land at distance* $6\sqrt{7}$ *m from A Minimum*

Example (18)

An iron factory produces two kinds of iron A and B. If the facory produces y tons from A and **40** *Example (18)*
An iron factory produces two kinds of iron A and B. If the facory produces y tons from A a
x tons from B such that $y = \frac{40 - 5x}{10 - x}$, where $x \ne 10$, if the price of each ton from kind A is $kinds$
 $\frac{40 - 5x}{10 - x}$
ton fre *equal twice the price of each ton from kind B. Then how many tons the factory produces from* − **Example (18)**
wo kinds of iron A and B. If the facory
= $\frac{40-5x}{10-x}$, where $x \ne 10$, if the price of *each kind to obtain a maximum profit .*

Answer

In this problem, we have to make a relation between the given equation and the price of each ton

each kind to obtain a maximum profit .
 Answer
 The factory produces **=** $A + B = y + x$ **
** *PH**IC''**is the price of each top from kind ''R'' Let "C" is the price of each ton from kind "B"*

factory produces = A + B = y + x

" is the price of each ton from kind "B"

total prices of the two kinds per each ton : $P = 2Cy + Cx = 2C\left[\frac{40 - x}{10 - x} + Cx \implies \therefore \frac{dP}{dx} = \frac{(10 - x)(-10C) - (80C - 10Cx)(-1)}{(10 - x)^2}\right]$ *4 this problem, we have to make a relation between the given equation and the price of*
The factory produces = $A + B = y + x$
zt "*C*" *is the price of each ton from kind "B"*
The total prices of the two kinds per each t $\frac{40 - 5x}{10 - x}$ *P* = $\frac{80C - 10Cx}{10 - x}$ + $Cx \Rightarrow C$. *al prices of the two kinds pe*
 $\frac{C-10Cx}{10-x} + Cx \Rightarrow \therefore \frac{dP}{dx}$ *Ln this problem, we have to make a relation between the given equation and the price of each ton*

∴ The factory produces = A + B = y + x
 Let "C" is the price of each ton from kind "B"

∴ The total prices of the two $\left[\frac{40-5x}{10-x}\right]+C x$ the price of each ton from kind "B"

prices of the two kinds per each ton : $P = 2Cy + Cx = 2C\left[\frac{40 - 5x^2}{10 - x}\right]$
 $\frac{-10Cx}{2-x} + Cx \implies \therefore \frac{dP}{dx} = \frac{(10 - x)(-10C) - (80C - 10Cx)(-1)}{(10 - x)^2} + C$: The factory produces = A + B = y + x

Let "C" is the price of each ton from kind "B"
 \therefore The total prices of the two kinds per each ton : $P = 2Cy + Cx = 2C\left[\frac{40 - 5x}{10 - x}\right] + Cx$
 $\therefore P = \frac{80C - 10Cx}{10 - x} + Cx \implies \therefore \frac{dP}{dx$ = 2C y + C x = 2C $\left[\frac{40-5x}{10-x}\right]$ +
 $\left(\frac{20C-10C}{10-x}\right)$ + C
 $\left(\frac{10-x}{10-x}\right)$ - 20C + C $\left(\frac{10-x}{10-x}\right)$ 2 $\left[10-x\right]$
 $+Cx \Rightarrow \therefore \frac{dP}{dx} = \frac{(10-x)(-10C) - (80C - 10Cx)(-1)}{(10-x)^2} + C$
 $\frac{20Cx + 80C - 10Cx}{(10-x)^2} + C = \frac{-20C}{(10-x)^2} + C = \frac{-20C + C(10-x)^2}{(10-x)^2}$ $\left[\frac{40-5x}{10-x}\right] + Cx$
 $\frac{x(-1)}{x} + C$
 $\frac{(10-x)^2}{(x-x)^2}$ $\frac{dE}{dx} = \frac{(10-x)^2 + C}{(10-x)^2} + C$
 $\frac{10Cx}{(10-x)^2} + C = \frac{-20C}{(10-x)^2} + C = \frac{-20C + C(10-x)^2}{(10-x)^2}$
 $(10-x)^2 = 0 \Rightarrow \therefore (10-x)^2 = 20$ \Rightarrow :. -20C + C(10 - x)² = 0
 $\sqrt{5}$ \Rightarrow x = 10 ± 2 $\sqrt{5}$ \Rightarrow $\frac{1}{\sqrt{5}}$
 \Rightarrow 40C
 $\sqrt{(10-x)^3}$ \Rightarrow So when $x \approx 3$ $C x = 2C \left[\frac{40 - 5x}{10 - x} \right]$
 $\frac{C - 10C x \left(-1 \right)}{2} + C$ *2* 2^2 $+ C = \frac{2}{(10-x)^2} + C = \frac{1}{(10-x)^2}$
 $2^2 + C(10-x)^2 = 0 \Rightarrow \therefore (10-x)^2 = 20$ *2* When $\frac{dP}{dx} = 0 \Rightarrow$ \therefore $-20C + C(10 - x)^2 =$
 \therefore $10 - x = \pm 2\sqrt{5} \Rightarrow x = 10 \pm 2\sqrt{5} \Rightarrow$

And $\therefore \frac{d^2P}{dx^2} = \frac{-40C}{(10 - x)^3} \Rightarrow$ So when \boxed{x} $2C y -$
 $- (80)$
 $10 - x$ $P = \frac{80C - 10Cx}{10 - x} + Cx \Rightarrow \therefore \frac{dP}{dx} = \frac{(10 - x)(-10C) - (80C - 10Cx)(-1)}{(10 - x)^2} -$
 $\frac{dP}{dx} = \frac{-100C + 10Cx + 80C - 10Cx}{(10 - x)^2} + C = \frac{-20C}{(10 - x)^2} + C = \frac{-20C + C(10 - x)}{(10 - x)^2}$ $\frac{(10-x)(-10C)-(10C)}{(10-x)}$ $P = \frac{80C - 10C x}{10 - x} + C x \implies \therefore \frac{dP}{dx} = \frac{(10 - x)(-10C) - (80C - 10C x)}{(10 - x)^2}$
 $\frac{dP}{dx} = \frac{-100C + 10C x + 80C - 10C x}{(10 - x)^2} + C = \frac{-20C}{(10 - x)^2} + C = \frac{-20C + C(10C x)}{(10 - x)^2}$ $\therefore \frac{dP}{dx} = \frac{-100C + 10Cx + 80C - 10Cx}{(10 - x)^2} + C = \frac{-20C}{(10 - x)^2} + C = \frac{-20C + C(10)}{(10 - x)^2}$

When $\frac{dP}{dx} = 0 \Rightarrow \therefore -20C + C(10 - x)^2 = 0 \Rightarrow (\div C) \Rightarrow \therefore (10 - x)^2 = 20$ *dx 10 − x 1* $\left(10 - x\right)^2$ $\left(10 - x\right)^2$ $\left(10 - x\right)^2$ $\left(10 - x\right)^2$
 Nhen $\frac{dP}{dx} = 0 \Rightarrow \therefore -20C + C(10 - x)^2 = 0 \Rightarrow \therefore (10 - x)^2 =$
 $\therefore 10 - x = \pm 2\sqrt{5} \Rightarrow x = 10 \pm 2\sqrt{5} \Rightarrow \frac{20}{\sqrt{5}} \Rightarrow x = 10 \pm 2\sqrt{5}$ $\Rightarrow x = \frac{20}{\sqrt{5}} \Rightarrow x = \frac{20}{\sqrt$ $= \pm 2\sqrt{5}$ \Rightarrow
 $\frac{d^2P}{dx^2} = \frac{-40C}{(10 - x)}$ + − $\frac{10C x}{c} + C x \Rightarrow \therefore \frac{dP}{dx} = \frac{(10 - x)(-10C) - (80C - 10C x)(-1)}{(10 - x)^2} + C$
 $\frac{+10C x + 80C - 10C x}{(10 - x)^2} + C = \frac{-20C}{(10 - x)^2} + C = \frac{-20C + C(10 - x)^2}{(10 - x)^2}$:. $P = \frac{80C - 10Cx}{10 - x} + Cx \implies$:. $\frac{dP}{dx} = \frac{(10 - x)(-10C) - (80C - 10Cx)(-1)}{(10 - x)^2}$

:. $\frac{dP}{dx} = \frac{-100C + 10Cx + 80C - 10Cx}{(10 - x)^2} + C = \frac{-20C}{(10 - x)^2} + C = \frac{-20C + C(10 - x)}{(10 - x)^2}$ $(-x)^2$ $(10-x)^2$ $(10-x)^2$ $\frac{100C + 10C x + 80C - 10C x}{(10-x)^2} + C = \frac{-20C}{(10-x)^2} + C = \frac{-20C + C(10-x)^2}{(10-x)^2}$
= 0 \Rightarrow :. $-20C + C(10-x)^2 = 0$ (+ C) \Rightarrow :. $(10-x)^2 = 20$ \Rightarrow \therefore -20C + C(10 - x)² = C
 $2\sqrt{5}$ \Rightarrow $x = 10 \pm 2\sqrt{5}$ \Rightarrow
 $=\frac{-40C}{(10-x)^3}$ \Rightarrow So when $x \leq$ ∴ $-20C + C(10-x)^2 = 0$ (÷ C) ⇒ ∴ $(10-x)^2 = 20$

⇒ $x = 10 \pm 2\sqrt{5}$ ⇒ $\frac{|\cdot x \approx 5.6 \text{ Or } x \approx 14.4|}{\cdot 0.00}$
 $\frac{10C}{-x}$ ⇒ So when $\frac{x \approx 5.6 \text{ ton}}{x \approx 5.6 \text{ ton}}$ ⇒ ∴ $\frac{d^2P}{dx^2} = \frac{-40C}{(10-x)^3} < 0$

40 - 5.6(5) 12 $Qr = x \approx 14.4$
 $\frac{d^2P}{dx^2} = \frac{-40C}{(10-x)^3} < 0$
 $\frac{(5)}{6} \approx \frac{12}{4.4} \approx 2.7$ tons $\frac{|x-3.0|^{10n}}{dx^{2}} \rightarrow \frac{40-5.6(5)}{dx^{2}} \approx \frac{12}{4.4}$
 $\frac{-40C}{(10-x)^{3}} > 0$ which is refused *2* 2^{-1} $(10)^3$ *2* 2^2 $(10-x)^3$ $(\div C) \Rightarrow$ \therefore $(10-x)^2 = 20$
 x \approx 5.6 Or $x \approx 14.4$

5.6 ton \Rightarrow \therefore $\frac{d^2P}{dx^2} = \frac{-40C}{(10-x)^3} < 0$ $\frac{r}{\left(x - x^2\right) \left(1 + \frac{a^2}{r^2}\right)}$
 $\frac{d^2 P}{dx^2} = \frac{-40 C}{(10 - x^2)}$ ∴ $10 - x = \pm 2\sqrt{5}$ $\Rightarrow x = 10 \pm 2\sqrt{5}$ $\Rightarrow \therefore x \approx 5.6$ *Or* $x \approx 14.4$

And $\therefore \frac{d^2P}{dx^2} = \frac{-40C}{(10 - x)^3}$ \Rightarrow *So when* $\boxed{x \approx 5.6$ *ton* $\Rightarrow \therefore \frac{d^2P}{dx^2} = \frac{-40C}{(10 - x)^3}$ < 0

Then the profit is maximum and in this *n* \Rightarrow $\therefore \frac{d^2 P}{dx^2} = \frac{1}{10}$
 $\frac{0 - 5.6 (5)}{10 - 5.6} \approx \frac{12}{4.4}$ *dx²* $(10-x)^3$
 *dx*² $(10-5.6)$
 *d*₂ $\frac{40-5.6(5)}{10-5.6} \approx \frac{12}{4.4} \approx 2.7$ *tons*
 And when $x \approx 14.4$ *ton* \Rightarrow $\therefore \frac{d^2P}{dx^2} = \frac{-40$ *in this case* : $y \approx \frac{40 - x^2}{10}$
 $\frac{d^2P}{dx^2} = \frac{-40C}{(10 - x)^3} > 0$ w ⇒ \therefore $(10-x)^2 = 20$
 $\frac{6}{5}$ Or $x \approx 14.4$
 $\Rightarrow \therefore \frac{d^2 P}{dx^2} = \frac{-40 C}{(10-x)^3} < 0$ − and in this case : $y \approx \frac{40 - 5.6(5)}{10 - 5.6} \approx \frac{1}{4}$
 $\Rightarrow \therefore \frac{d^2 P}{dx^2} = \frac{-40C}{(10 - x)^3} > 0$ which is refu.

Example (19)

Let the required point be N, $NA = x \Rightarrow \therefore NB = 50 - x$
 Let the required point be N, $NA = x \Rightarrow \therefore NB = 50 - x$ *If AB = 50 cm and two light sources, one is put at A and the other is put at B. Find the point on
AB at which the light intensity is minimum given that the ratio between the light intensities of t AB at which the light intensity is minimum, given that the ratio between the light intensities of the* = *the distance between the source and this point .*

Answer

the distance between the source and this point.
\nLet the required point be N, NA =
$$
x \Rightarrow \therefore NB = 50 - x
$$

\nAnd $\therefore I_1 \alpha \frac{1}{x^2} \Rightarrow So \quad I_2 \alpha \frac{1}{(50 - x)^2}$
\n $\therefore I_1 = \frac{27k}{x^2} \Rightarrow I_2 = \frac{8k}{(50 - x)^2}$
\nAnd \therefore point N is affected by the two intensity light sources
\n $\therefore I_N = \frac{27k}{x^2} + \frac{8k}{(50 - x)^2} = 27kx^2 + 8k(50 - x)^2$

$$
\therefore I_1 = \frac{27k}{x^2} \implies I_2 = \frac{8k}{(50 - x)^2}
$$

\nAnd \therefore point N is affected by the two intensity light sources
\n $\therefore I_N = \frac{27k}{x^2} + \frac{8k}{(50 - x)^2} = 27kx^2 + 8k(50 - x)^2$
\n $\therefore I_{N'} = -54kx^3 - 16k(50 - x)^{-3}(-1) = \frac{-54k}{x^3} + \frac{16k}{(50 - x)^3}$
\nAnd \therefore I is minimum when $I_{N'} = 0 \implies \frac{-54k}{x^3} + \frac{16k}{(50 - x)^3} = 0$
\n $\therefore \frac{16k}{(50 - x)^3} = \frac{54k}{x^3} \implies \frac{8}{(50 - x)^3} = \frac{27}{x^3} \implies \left(\frac{2}{50 - x}\right)^3 = \left(\frac{3}{x}\right)^3$
\n $\therefore \frac{2}{50 - x} = \frac{3}{x} \implies \therefore 2x = 150 - 3x \implies \therefore 5x = 150 \implies \frac{1}{x} \implies \frac{x}{x} = 30 \text{ cm}$
\nExample (20)
\nA function f is defined by $f(x) = 9 - x^2$ where $0 \le x \le 3$. A is a point on the curve off, O is the origin point. \overline{AB} and \overline{AC} are perpendiculars drawn from A to the x - axis and y - axis respectt

Example (20)

origin point. AB and AC are perpendiculars drawn from A to the x - axis and y - axis respectively. Find the coordinates of A in order that the surface area of the rectangle ABOC should be maximum

Answer

In this problem, we have to make a relation between the required point area of the rectangle
Let the dimension of the rectangle be x and $y \Rightarrow \therefore y = 9 - x^2 - (-1)^n$ *In this problem, we have to make a relation between the required point and the* area of the rectangle
Let the dimension of the rectangle be x and $y \Rightarrow$ $\therefore y = 9 - x^2 - (-1)$ *area of the rectangle*

-3 0 3 g \rightarrow *A*(*x,y*) *B C x y* $\frac{int \text{ and } the}{\sqrt{\frac{1}{\text{odd}}}}$ $\frac{x}{x}$
 $\frac{y}{y} \Rightarrow \frac{y}{y} = \frac{y-2}{x^2 - (-1)^2}$
 $\left(9-x^2\right) = 9x - x^3$
 $\frac{9}{x^2} = 0 \Rightarrow 3x^2 = 9$ $\begin{array}{c}\n\begin{array}{c}\n\cdot & y - 9 - x & - - - (1) \\
x^2 & = 9x - x^3\n\end{array}\n\end{array}\n\qquad\n\begin{array}{c}\n\hline\n9 \\
0 \Rightarrow 3x^2 = 9 \\
\hline\n(\text{refused})\n\end{array}\n\qquad\n\begin{array}{c}\nC \\
\hline\n\end{array}$ $\int \csc x \, dx$
 $\int \csc x \, dx$
 $\int 3x^2 \implies \text{ when } A' = 0 \implies 9$
 \implies so, either $\boxed{x = \sqrt{3}}$ or \boxed{x}
 $\left(\sqrt{3}\right)^2 = 6$, then the coording $\begin{aligned} \therefore y &= 9 - x^2 \\ \therefore y &= 9x - x^3 \end{aligned}$ \therefore $A' = 9 - 3x^2 \implies$ when $A' = 0 \implies 9 - 3x^2 = 0 \implies 3x^2 = 9$ *2 2 And : the area of the rectangle be x and y* \Rightarrow $\boxed{\because y = 9}$ *
And : the area of the rectagle A = x y = x* $\left(9 - x^2\right) = 9x - x^2$ Let the dimension of the rectangle be x and $y \Rightarrow$ $\therefore y = 9 - x^2 - (-1)$
And \therefore the area of the rectagle $A = x$ $y = x(9 - x^2) = 9x - x^3$
 $\therefore A' = 9 - 3x^2 \Rightarrow \text{ when } A' = 0 \Rightarrow 9 - 3x^2 = 0 \Rightarrow 3x^2 = 9$ *nd* : *the area of the rectagle* $A = x$ $y = x(9 - x^2) = 9x$
 $A' = 9 - 3x^2 \implies when A' = 0 \implies 9 - 3x^2 = 0 \implies 3$
 $x^2 = 3 \implies so, either \boxed{x = \sqrt{3}}$ or $\boxed{x = -\sqrt{3}}$ (refused $A = 9-3x \rightarrow$ when
 $x^2 = 3 \Rightarrow$ so, either $\left[\frac{y}{x}\right]^2 = 6$, the be x and $y \implies$ $\boxed{\because y = 9 - x^2 - (-1)^2}$
= x $y = x(9 - x^2) = 9x - x^3$ And : the area of the rectagle $A = x$ $y = x(9 - x^2) = 9$
 $\therefore A' = 9 - 3x^2 \implies when A' = 0 \implies 9 - 3x^2 = 0 \implies$
 $\therefore x^2 = 3 \implies so, either \boxed{x = \sqrt{3}} \text{ or } \boxed{x = -\sqrt{3} \text{ (refuse)}}$ And : the area of the rectangle be x and y \Rightarrow $\frac{y}{x} = \frac{y}{x}$ (x)

And : the area of the rectagle $A = x$ $y = x(9 - x^2) = 9x - x^3$
 \therefore $A' = 9 - 3x^2 \Rightarrow$ when $A' = 0 \Rightarrow 9 - 3x^2 = 0 \Rightarrow 3x^2 = 9$
 \therefore $x^2 = 3 \Rightarrow$ so, either $\boxed{x$

Example (21)

k equals 20 πa^2 . Prove that if the volume of the tank is maximum, then the length of the b

dius of the cylinder equals its height .
 Answer
 The total area of the cylinder = Area of the two bases + (perimeter of *A water tank is in the shape of a cylinder above which there is a semi - sphere such that the upper* base of the cylinder is itself the plane surface of the semi - sphere. If the total surface area of the πa^2 *Example (21)*
A water tank is in the shape of a cylinder above which there is a semi - sphere such that the uppo
base of the cylinder is itself the plane surface of the semi - sphere. If the total surface area of the radius of the cylinder equals its height .

Answer

(*perimeter of the base* \times *height*) = Area of the two bases $+$ (perimeter of the base \times

() *² ² ² 2 2 2 2 1 r 2 r h 4 r 20 a 20 a 3r 2rh 2 20a 3r h ¹ 2 r* + + = = + [−] = − − − *Area of the base height Volume of the semi - sphere* +

 Volume of the total cylinder :

$$
\therefore h = \frac{20a^2 - 3r^2}{2r} - (-1)
$$

\n
$$
\therefore \text{ Volume of the total cylinder :}
$$

\n
$$
\therefore V = \pi r^2 h + \frac{1}{2} \times \frac{4}{3} \pi r^3 = \pi r^2 h + \frac{2}{3} \pi r^3 - (-2)
$$

\nBy substituting (1) in (2): $V = \pi r^2 \left[\frac{20a^2 - 3r^2}{2r} \right] + \frac{2}{3} \pi r^3$
\n
$$
\therefore V = \frac{\pi}{2} (20a^2 r - 3r^3) + \frac{2}{3} \pi r^3 \implies V' = \frac{\pi}{2} (20a^2 - 9r^2) + 2\pi r^2
$$

g (*1*) *in* (2) \therefore $V = \pi r^2 \left[\frac{26a^2 - 3r^2}{2r} \right] + \frac{2}{3} \pi r^3$
 $V' = \frac{\pi}{2} (20a^2 - 9r^2) + 2\pi r^2$ *2* 3
 V substituting (1) *in* (2) \therefore $V = \pi r^2 \left[\frac{20a^2 - 3r^2}{2r} \right] + \frac{2}{3} \pi r^3$
 $V = \frac{\pi}{2} (20a^2r - 3r^3) + \frac{2}{3} \pi r^3 \implies V' = \frac{\pi}{2} (20a^2 - 9r^2) + 2\pi r^2$ $\pi r^2 h + \frac{1}{2} \times \frac{1}{3} \pi r^3 = \pi r^2 h + \frac{1}{3} \pi r^3 = -(-2)$
stituting (1) in (2) : $V = \pi r^2 \left[\frac{20a^2 - 3r^2}{2r} \right] + \frac{2}{3} \pi r^3$
 $\frac{\pi}{2} (20a^2 r - 3r^3) + \frac{2}{3} \pi r^3 \Rightarrow V' = \frac{\pi}{2} (20a^2 - 9r^2) + 2$: $V = \pi r^2 \left[\frac{20a^2 - 3r^2}{2r} \right] + \frac{2}{3} \pi r^3$
 $\pi r^3 \Rightarrow V' = \frac{\pi}{2} (20a^2 - 9r^2) + 2\pi r^2$ By substituting (1) in (2)∴ $V = \pi r^2 \left[\frac{20a^2 - 3r^2}{2r} \right] + \frac{2}{3} \pi r^3$
∴ $V = \frac{\pi}{2} (20a^2r - 3r^3) + \frac{2}{3} \pi r^3$ ⇒ $V' = \frac{\pi}{2} (20a^2 - 9r^2) + 2\pi r^2$

$$
\therefore V = \frac{\pi}{2} (20a^2r - 3r^3) + \frac{2}{3}\pi r^3 \implies V' = \frac{\pi}{2} (20a^2 - 9r^2) + 2\pi r^2
$$

And :: the v $(20a^2 - 9r^2) + 2$
 $(2a^2 - 9r^2) + 2\pi r^2$ $(a^2r - 3r^3) + \frac{2}{3}\pi r^3 \Rightarrow V' = \frac{\pi}{2}(20a^2 - 9r^2) + 2\pi r^2$
 olume is maximum when $\frac{\pi}{2}(20a^2 - 9r^2) + 2\pi r^2 = 0$ $\begin{aligned}\n\int_{a^2 - 9r^2}^{+3} dx \\
-9r^2 + 2\pi r^2 \\
-9r^2 + 2\pi r^2 &= 0\n\end{aligned}$

By substituting (1) in (2):
$$
V = \pi r^2 \left[\frac{20a^2 - 3r^2}{2r} \right] + \frac{2}{3}\pi r^3
$$

\n $\therefore V = \frac{\pi}{2} \left(20a^2r - 3r^3 \right) + \frac{2}{3}\pi r^3 \implies V' = \frac{\pi}{2} \left(20a^2 - 9r^2 \right) + 2\pi r^2$
\nAnd \therefore the volume is maximum when $\frac{\pi}{2} \left(20a^2 - 9r^2 \right) + 2\pi r^2 = 0$
\n $\therefore 10a^2 = \frac{5}{2}r^2 \implies r^2 = 10a^2 \times \frac{2}{5} = 4a^2 \implies \frac{7}{6} \therefore r = 2a$
\nFrom (1): $h = \frac{20a^2 - 12a^2}{4a} = \frac{8a^2}{4a} = 2a \implies \frac{7}{6} \therefore r = h$

h

Example (22)

Find the greatest area of an isosceles triangle can be drawn inside a circle of raduis 15 cm

Answer

2 Let x be the distance between the center of the circle and the base of the isosceles tind the greatest area of an isosceles triangle can be drawn insid
Answer
*Let x be the distance between the center of the circle and the base
<i>triangle, then the base of the triangle* $2\sqrt{225 - x^2}$ *and the height*
*o <u>Answerent 2</u>
Let x be the distance between the center of the circ
triangle, then the base of the triangle* $2\sqrt{225-x^2}$ *
of the triangle is* $15 + x$ *, therefore, the area of the t*</u> − + *riangle is given by* the distance between the center of the circ
gle, then the base of the triangle $2\sqrt{225 - x^2}$
e triangle is $15 + x$, therefore, the area of the t
 $=\frac{1}{2}(2\sqrt{225 - x^2})(15 + x)$ *angle, then the base of the i*
the triangle is $15 + x$ *there*
 $A = \frac{1}{2} \left(2\sqrt{225 - x^2} \right) \left(15 + x \right)$

Example (23)

Find the area of the largest trapezoid that can be inscribed in a circle of radius 1 unit and
whose base is the diameter of the circle. *Find the area of the largest trapezoid the*
whose base is the diameter of the circle.

Then the maximum area of the trapezoid occurs when
$$
x =
$$

is $A = \left(I - \left(\frac{I}{2}\right)^2 \right)^{\frac{1}{2}} \left(\frac{I}{2} + I\right) = \frac{3\sqrt{3}}{4}$ square unit

Example (24)

The cost of the fuel consumption of a locomotive is proportional to the square of its speed. The cost of the fuel consumption is 25 pounds per hour when the speed is 25 km / hr. There is 25 km / hr. There
notive speed . Find the
 $(25)^2$

is also an additional cost 100 pounds per hour regardless of the locomotive speed. Find the
\nspeed of the locomotive which minimizes the total cost of one kilometer.
\nLet the cost per hr. is
$$
T \Rightarrow \therefore T \alpha v^2 \rightarrow T = kv^2 \Rightarrow \therefore 25 = k(25)^2
$$

\n $\therefore k = \frac{1}{25} \Rightarrow \therefore \boxed{T = \frac{1}{25}v^2 \ L.E/hr}$
\n \therefore There is an additional cost 100 per hour: $\therefore C = \frac{1}{25}v^2t + 100t$
\n \therefore Time = $\frac{distance(D)}{velocity(V)} \Rightarrow after (1 km) \Rightarrow \therefore T = \frac{1}{V} hr$
\n $\therefore C = \frac{1}{25}v^2 \times \frac{1}{v} + 100 \times \frac{1}{v} = \frac{1}{25}v + \frac{100}{v} \Rightarrow \therefore \frac{dc}{dv} = \frac{1}{25} - \frac{100}{v^2}$
\nAnd $\therefore C$ is minimum when $\frac{dc}{dv} = 0 \Rightarrow \therefore \frac{1}{25} - \frac{100}{V^2} = 0 \Rightarrow \therefore \frac{100}{V^2} = \frac{1}{25}$
\n $\therefore V^2 = 2500 \Rightarrow \therefore V = 50 km/hr$

Example (25)

 $point\ B\ (on\ the\ natural\ exponential\ curve)\ and\ point\ C$ *In the opposite figure: a point A is moving in the positive direction of x - axis starting from* $(0,0)$, *find the value of x such that the distance between* (*on the natural logarithmic curve*) *are smallest as possible .*

Answer

Let S is the distance between the two curve where :

 $X \sim \text{Im } x \rightarrow \text{Im } S' = e^{x}$ $X \rightarrow \text{Im } S' = 0 \rightarrow \text{Im } x \rightarrow \text{Im } S' = 0 \rightarrow \text{Im } x \$ $\frac{x}{t}$ $\frac{1}{t}$ $\frac{x}{t}$ *Answer*
Let S is the distance between the two curve where :
 $S = e^x - \ln x \implies$ $\therefore S' = e^x - \frac{1}{x} \implies \text{when } S' = 0 \implies \therefore e^x = \frac{1}{x} \implies \boxed{\therefore e^x = x^{-1}} \implies \ln \text{ both sides}$ *Answer*
 x x x x \Rightarrow *when S'* = 0 \Rightarrow $\therefore e^{x} = \frac{1}{x}$
 x $\Rightarrow e^{x} = -\ln x$ \Rightarrow $\therefore \ln x = -x$ $\Rightarrow \infty$ *ln s* is the distance between the two curve where :
 $e^x - ln x \implies$ $\therefore S' = e^x - \frac{1}{x} \implies when S' = 0 \implies \therefore e^x = \frac{1}{x} \implies \boxed{\therefore e^x = x^{-1}} \implies ln \text{ both sides}$
 $ln e^x = ln x^{-1} \implies \therefore x ln e = -ln x \implies \boxed{\therefore ln x = -x} \implies can't be solved except by calculator.$ $e^{x}-\ln x \Rightarrow$ \therefore $S' = e^{x}-\frac{1}{x} \Rightarrow$ when $S' = 0 \Rightarrow$
 $\ln e^{x} = \ln x^{-1} \Rightarrow$ \therefore $x \ln e = -\ln x \Rightarrow$ $\boxed{\therefore \ln x} \Rightarrow$
 $\boxed{\ln x} \rightarrow \boxed{Alpha} \rightarrow \boxed{Calc} \rightarrow \boxed{-x} \rightarrow \boxed{Shift}$ − *Answer*
 et S is the distance between the two curve where :
 $= e^x - \ln x \Rightarrow$ ∴ $S' = e^x - \frac{1}{x} \Rightarrow$ when $S' = 0 \Rightarrow$ ∴ $e^x = \frac{1}{x} \Rightarrow$ $\boxed{\therefore e^x = x^{-1}} \Rightarrow$ In both sides Let S is the distance between the two curve where :
 $S = e^x - \ln x \implies$ $\therefore S' = e^x - \frac{1}{x} \implies$ when $S' = 0 \implies$ $\therefore e^x = \frac{1}{x} \implies$ $\therefore e^x = \frac{1}{x} \implies$ $\therefore e^x = \frac{1}{x} \implies e^x = \frac{1}{x} \implies e^x = \frac{1}{x} \implies e^x = \frac{1}{x} \implies e^x = \frac{1}{x} \implies$ $S = e^x - \ln x \implies$ $\therefore S' = e^x - \frac{1}{x} \implies$ when $S' = 0 \implies$ $\therefore e^x = \frac{1}{x} \implies$ $\boxed{\therefore e^x = x^{-1}} \implies \ln b$
 $\therefore \ln e^x = \ln x^{-1} \implies$ $\therefore x \ln e = -\ln x \implies \boxed{\therefore \ln x = -x} \implies$ can't be solved except by co
 $\therefore \boxed{\ln x} \implies \boxed{\text{Alpha}} \implies \boxed{\text{Calc}} \implies \boxed{-x} \impl$ $x + \frac{1}{x^2} > 0$ [minimum at x = 0.567] *Solutional A* \therefore *Solutional A* \therefore *Solutional A* \Rightarrow *Solutional A* \Rightarrow *Solutional* \Rightarrow *Solutional* \Rightarrow *Solutional* \Rightarrow *Solutional A* \Rightarrow *Solutional A* \Rightarrow *Solutional A* \Rightarrow *Solutional A* \Rightarrow *Solu*

Revision on the previous year on Integration

Remember that

Rules

EXECUTE: The function is the value of the reverse of differentiation.
\n(1)
$$
\int x^n dx = \frac{x^{n+1}}{n+1} + c
$$
, where $n \ne -1$, and c is a constant
\n $\boxed{\ln words}$ Add (1) to the power and divide by the new power
\n(2) $\int dx = \int 1 dx = x + c$
\n(3) $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$
\n $\boxed{\ln words}$ $\frac{Add(1) to the power}{(New power)(derivative between the bracket)}$

Note There is no general rule for finding the integration of multiplication or division of two functions .

Find the integral of each of the following functions :

Remember that Integration is the undo or the reverse of differentiation.
\n**Rules**
\n(i)
$$
\int x^n dx = \frac{x^{n+1}}{n+1} + c
$$
, where $n \ne -1$, and c is a constant
\n $\frac{[In words]}{[In words]} Add(1)$ to the power and divide by the new power
\n(2) $\int dx = \int 1 dx = x + c$
\n(3) $\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + c$
\n $\frac{[In words]}{[Neov power]} (Adi(1))$ to the power
\nAdd (1) to the power
\nAdd (1) to the power
\nAdd (2) If $ax = b$ if $ax = b$ and $ax = b$
\n $\frac{[In words]}{[Neov power]} (Aerivative between the bracket)$
\n $\frac{[In words]}{[In words]} = \frac{[In words]}{[Neov power]} (Aerivative of multiplication or division of the functions.)$
\n $\frac{[In words]}{[In words]} = \frac{[In words]}{[Neoving functions]} = \frac{[In words]}{[In words]} = \frac{[In words]}{[In words$

Integration of basic trigonometric functions

Examples
 Examples
 Examples
 Examples
 Examples E
Remember the following
 $\frac{[e]}{(1) : \operatorname{Sin}^2 x + \operatorname{Cos}^2 x}$
 $\frac{(2) : 1 + \operatorname{Tor}^2 x - \operatorname{Sec}}{(2) : 2 + \operatorname{Cox}^2 x}$ E Remember the follo

(1) : $Sin^2x + Cos^2x =$

(2) : $1 + Tan^2x = Sec$ (1): $\sin^2 x + \cos^2 x = 1 \implies$ $\therefore \cos^2 x = 1 - \sin^2 x$ or Sin(2): $1 + \tan^2 x = \sec^2 x \implies$ $\therefore \tan^2 x = \sec^2 x - 1$ or So
(3) $\sin 2x = 2 \sin x \cos x$ (4) $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$ (2): $1 + Tan^2x = Sec$

(3) $Sin 2x = 2Sin xCc$

(5) $Cos 2x = Cos^2x -$ *2*
 2 $x + Cos^2x = 1 \implies$ $\therefore Cos^2x = 1 - Sin^2x$ or $Sin^2x = 1 - Cos^2x$ $\overline{Cos^2x} = 1 \implies$ $\therefore \ \overline{Cos^2x} = 1 - \overline{Sin^2x} \quad \text{or} \quad \overline{Sin^2x} = 1 - \overline{Cos^2x}$
 $\overline{C}^2x = \overline{Sec}^2x \implies$ $\therefore \ \overline{Tan^2x} = \overline{Sec}^2x - 1 \quad \text{or} \quad \overline{Sec}^2x - \overline{Tan^2x}$ $2x \cos x$
 $2x - \sin^2 x$ $2cos^2 x - sin^2 x = 2Cos^2 x - 1 = 1 - 2Sin^2 x$ *2 Remember the following relations between the trigonometric functions*
 1): $\sin^2 x + \cos^2 x = 1 \implies$ $\therefore \cos^2 x = 1 - \sin^2 x$ or $\sin^2 x = 1 - \cos^2 x$ *2* 1 The *Pollocal Bure 1 Tan²x + Cos²x = 1*
2 1 Tan²x = Sec²x *I*): $\sin^2 x + \cos^2 x = 1 \implies \therefore \cos^2 x = 1 - \sin^2 x$ or Sin 2): $1 + \tan^2 x = \sec^2 x \implies \therefore \tan^2 x = \sec^2 x - 1$ or Son 3) $\sin 2x = 2\sin x \cos x$ (4) $\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$ $\frac{2}{x}$
 $2x$ *1 or Sec*
 $\frac{2Tan x}{1 - Tan^2x}$ *5 Cos2x Cos x S in x 2Cos x 1 ¹ Sin x* As a result of that: *Se Tos²x* = 1 - *Sin²x* or *Sin²x* = 1 - *Cos²x*
Tan²x = *Sec²x* - 1 or *Sec²x* - *Tan²x* = 1 *or o r r* the following relations between the trigonometric functions
+ $Cos^2 x = 1 \implies$ $\therefore Cos^2 x = 1 - Sin^2 x$ or $Sin^2 x = 1 - Cos^2 x$ $\frac{1 - \cos^2 x}{-1 - \tan^2 x}$ $\ddot{\cdot}$ mber the following relations between the trigonometric
 $\frac{\sin^2 x + \cos^2 x = 1 \Rightarrow \therefore \cos^2 x = 1 - \sin^2 x \text{ or } \sin^2 x = 1}{1 - \sin^2 x}$
 $+ \tan^2 x = \sec^2 x \Rightarrow \therefore \tan^2 x = \sec^2 x - 1 \text{ or } \sec^2 x$ = ∴ Ian $x = \sec x - 1$ or $\sec x$

(4) $Tan 2x = \frac{2Tan x}{1 - Tan^2 x}$

= $2 Cos^2 x - 1 = 1 - 2Sin^2 x$ = \Rightarrow \Rightarrow − *2 2* (4) I *an* $2x =$
 $2Cos^2x - 1 = 1 - 2Si$
 $\overline{Sin^2x} = \frac{1}{2} - \frac{1}{2}Cos2x$ $\frac{l}{\frac{l}{2} - \frac{l}{2}}$ $\overline{Sin^2x} = \frac{1}{2} - \frac{1}{2}Cos2x$
 $\overline{Cos^2x} = \frac{1}{2} + \frac{1}{2}Cos2x$ *2 2* $I-T$
 $= I - 2Sin^2x$
 $= \frac{I}{2} - \frac{I}{2}Cos 2x$ $=\frac{1}{2} - \frac{1}{2} \cos 2x$
= $\frac{1}{2} + \frac{1}{2} \cos 2x$

Example (1)

Find: $\int (Sin^{-2}2x + Cos^{-2}2x)dx$

Example (1)
\nFind:
$$
\int (\sin^2 2x + \cos^2 2x) dx
$$
\n
$$
\therefore \sin^2 2x + \cos^2 2x = 1 \implies \therefore \int (\sin^2 2x + \cos^2 2x) dx = \int dx = x + c
$$

 \therefore $\sin^2 2x + \cos^2 2x = 1 \implies \therefore \int (\sin^2 2x + \cos^2 2x) dx = \int dx = x + c$
Example (2)
Find: $\int \sin 3x \cos 3x dx$ Answer $\int \frac{1}{2} \sin 6x dx = \frac{-1}{12} \cos 6x + c$ *Example (2) 1 Answer* $\int \frac{1}{2}$ *Example (2)*
Sin3xCos3x dx <u>*Answer*</u> $\int \frac{1}{2}$ *Sin 6x dx* = $\frac{-1}{12}$ *Cos 6x+c 2 1 dx* <u>Answer</u> *2*
 Example (2)
 Find : $\int \sin 3x \cos 3x \, dx$ <u>*Answer*</u> $\int \frac{1}{2} \sin 6x \, dx = \frac{-1}{12} \cos 6x + c$

Examples

Example (1)

2 1 Find : $\int \frac{1}{1 - \cos^2 x} dx$

 $\frac{\cos x}{\cos^2 x}dx$

Answer

2 $\int \frac{1}{1 - \cos^2 x} dx$
 2 $\int \frac{1}{\sin^2 x} dx = \int \csc^2 x dx = -\cot x + c$ $\frac{1}{1 - \cos^2 x} dx = \int \frac{1}{\sin^2 x} dx$ $\frac{1}{-Cos^2x}$ $\frac{1}{-\cos^2x}$ $dx = \int \frac{1}{\sin^2x} dx = \int Cosec^2x dx = -\cot x + c$ Find: $\int \frac{I}{I - Cos^2 x} dx$
 $\int \frac{I}{I - Cos^2 x} dx = \int \frac{I}{Sin^2 x} dx = \int Cosec^2 x dx = -$

Notes $Sin^{2}x + Cos^{2}x = 1$ \therefore *Sin²x* = 1 – Cos^2x

Example (2)

Find : $\int \frac{Cos x}{1 - Cos^2 x}$ *Answer 2 2 Cos x Cos x Cos x 1 dx dx dx Cosec xCot x dx -Cosec x c nd*: $\int \frac{\cos x}{1 - \cos^2 x} dx$
 $\int \frac{\cos x}{\sin^2 x} dx = \int \frac{\cos x}{\sin x} dx = \int \frac{\cos x}{\sin x} dx$ d : $\int \frac{Cos x}{1 - Cos^2 x} dx$
 $\frac{Loss x}{1 - Cos^2 x} dx = \int \frac{Cos x}{Sin^2 x} dx = \int \frac{Cos x}{Sin x} \times \frac{1}{Sin x} dx = \int Cosec x \cdot Cost x dx = -Cosec x + c$ Example (2)

Find: $\int \frac{Cos x}{1 - Cos^2 x} dx$
 $\int \frac{Cos x}{1 - Cos^2 x} dx = \int \frac{Cos x}{Sin^2 x} dx = \int \frac{Cos x}{Sin x} \times \frac{1}{Sin x} dx = \int Cos e x \cdot Cot x dx = -Cot$.

Example (3)

 $\frac{2}{\pi} \left(\frac{x+3}{2} \right) dx$ *2* $\left(\frac{x+3}{x}\right)_{dx}$ *Find :* $\int \text{Cosec}^2 \left(\frac{x+3}{2} \right) dx$ *Answer* $\int \csc^2\left(\frac{x}{2}\right)dx$
 $\int \csc^2\left(\frac{1}{2}x + \frac{3}{2}\right)dx$ *2 1 2 <i>1 2 <i>niswer*
Cosec² $\left(\frac{1}{2}x + \frac{3}{2}\right)dx = \frac{-Cot\left(\frac{1}{2}x + \frac{3}{2}\right)}{\frac{1}{2}} + c = -2Cot\left(\frac{1}{2}x + \frac{3}{2}\right) + c$ $\frac{1}{2}x + \frac{3}{2}dx = \frac{-Cot(\frac{1}{2}x + \frac{3}{2})}{\frac{1}{2}} + c = -2Cot(\frac{1}{2}x + \frac{3}{2})$ *2* $\frac{Answer}{2}$ $\left(\frac{1}{2}x + \frac{3}{2}\right) + c = -2C_0$ $\int \text{Cosec}^2 \left(\frac{1}{2} x + \frac{3}{2} \right) dx = \frac{-\text{Cot} \left(\frac{1}{2} x + \frac{3}{2} \right)}{\frac{1}{2}} + c = -2 \text{Cot} \left(\frac{1}{2} x + \frac{3}{2} \right) + c$

Example (6)

Find:
$$
\int \frac{Sin^{2}(2x-3)}{1-Cos(2x-3)} dx
$$

$$
\int \frac{1-Cos^{2}(2x-3)}{1-Cos(2x-3)} dx = \int \frac{1-Cos(2x-3)}{1-Cos(2x-3)} \frac{Answer}{1-Cos(2x-3)} dx = \int 1+Cos(2x-3) dx
$$

$$
= x + \frac{Sin(2x-3)}{2} + c
$$

Example (7)

Find: $\int Tan^2x + 2Sin^2x dx$

Answer

$$
\int (Sec^{2}x - I) + 2\left(\frac{1}{2} - \frac{1}{2}Cos 2x\right) dx = \int Sec^{2}x - I + I - Cos 2x \ dx = \int Sec^{2}x - Cos 2x \ dx
$$

= $Tan x - \frac{1}{2}Sin 2x + c$

Integration forms

Examples

Find the integration of the following integrals

Example (1)

$$
\int \frac{x^5 - 3x^3 + 2}{x^2}
$$

$$
\int \frac{x^5 - 3x^3 + 2}{x^2}
$$

By using algebra: :
$$
\int \frac{x^5 - 3x^3 + 2}{x^2} dx = \int \frac{x^5}{x^2} - \frac{3x^3}{x^2} + \frac{2}{x^2} dx
$$

.:
$$
\int (x^3 - 3x + 2x^{-2}) dx = \frac{1}{4}x^4 - \frac{3}{2}x^2 - 2x^{-1} + c
$$

Example (2)

 $\frac{16x^4 - 81}{2}$ dx $2x + 3$ − $\int \frac{10x}{2x+}$

Answer

By using algebra:

Example (2)
\n
$$
\int \frac{16x^4 - 81}{2x + 3} dx
$$
\n
\nBy using algebra:
\n
$$
\int \frac{(4x^2 + 9)(2x - 3)(2x + 3)}{(2x + 3)} dx = \int (4x^2 + 9)(2x - 3) dx = \int (8x^3 - 12x^2 + 18x - 27) dx
$$
\n
$$
= \frac{8x^4}{4} - \frac{12x^3}{3} + \frac{18x^2}{2} - 27x = 2x^4 - 4x^3 + 9x^2 - 27x + c
$$

Example (3)

 $\frac{8x^3 + 27}{2} dx$ $2x + 3$ +

Answer

By using algebra:

$$
\int \frac{8x^3 + 27}{2x + 3} dx
$$

By using algebra :

$$
\int \frac{(2x + 3)(4x^2 - 6x + 9)}{2x + 3} dx = \int (4x^2 - 6x + 9) dx = \frac{4}{3}x^3 - 3x^2 + 9x + c
$$

Example (4)

 $\frac{x-4}{x}$ dx $x + 2$ − $\int \frac{x}{\sqrt{x+}}$

Answer

 $\sqrt{x} + 2$ ^{axi}
By using algebra :

$$
\int \frac{x-4}{\sqrt{x}+2} dx
$$

By using algebra :

$$
\int \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(\sqrt{x}+2)} dx = \int (\sqrt{x}-2) dx = \frac{2}{3}x^{\frac{3}{2}} - 2x + c
$$

Example (5)

 $x^5\left(1-\frac{1}{x}\right)^5 dx$ $\left(1-\frac{1}{x}\right)^5c$ $\int x^5 \left(1 - \frac{1}{x}\right) dx$

Answer

By using algebra :

By using algebra :
\n
$$
\int \left[x \left(1 - \frac{1}{x} \right) \right]^5 dx = \left(x - 1 \right)^5 dx = \frac{1}{6} \left(x - 1 \right)^6 + c
$$

$$
\begin{array}{ll}\n\text{using algebra:} \\
x\left(1-\frac{1}{x}\right)\end{array} dx = (x-1)^5 dx = \frac{1}{6}(x-1)^6 + c
$$
\n
$$
\begin{array}{|l|}\n\hline\n(6) \int e^{7x} dx & (7) \int 3\sqrt{2} e^{-\sqrt{2}n} dn & (8) \int 8 e^{\frac{-3}{4}y} dy \\
\hline\n\frac{Answer}{7} + c & \frac{3\sqrt{2} e^{-\sqrt{2}n}}{-\sqrt{2}} + c = -3 e^{-\sqrt{2}n} + c \\
\hline\n(9) \int \frac{2}{x} dx & (10) \int \frac{7}{x \ln 3} dx & (11) \int x^{2e} + e^{3x} dx \\
\hline\n\frac{Answer}{4} & 2 \int \frac{1}{x} dx = 2 \ln|x| + c & \frac{7}{\ln 3} \int \frac{1}{x} \frac{1}{\ln 5} + \frac{7}{x} \ln|x| + c \\
\hline\n\end{array}
$$

Example (12)

2

Ē.

Î.

i.

Example (12)
\n
$$
\int \frac{\ln x^2}{x \ln x^3} dx
$$
\n
$$
\frac{\text{Answer}}{\text{Number: } \int \frac{2 \, y \, x}{3 \, x \, y \, x^2} \, dx} = \int \frac{2}{3x} \, dx - \frac{By \, using \, bundle}{3} \, y \, \frac{2}{x} \int \frac{1}{x} \, dx = \frac{2}{3} \ln |x| + c
$$
\n
$$
\int \frac{3e^x - 2e^{2x}}{2e^x} \, dx
$$
\n
$$
\frac{1}{2} \int \frac{3e^x - 2e^{2x}}{e^x - e^x} \, dx = \frac{1}{2} \int 3 - 2e^x \, dx = \frac{1}{2} \left[3x - 2e^x \right] + c = \frac{3}{2}x - e^x + c
$$
\n
$$
\int e^{\int \tan x} \, \sec^2 x \, dx
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\int e^{\int \tan x} \, \sec^2 x \, dx
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Example (18)

Tan x dx

Answer

$$
\int \tan x \, dx
$$
\n
$$
\int \frac{\sin x}{\cos x} \, dx = -\int \frac{-\sin x}{\cos x} \, dx = -\ln |\cos x| + c = \ln |(\cos x)^{-1}| + c = \ln |\sec x| + c
$$
\n
$$
\int \frac{f'(x)}{f'(x)} \, dx = -\ln |\cos x| + c = \ln |\cos x| + c
$$

Example (19)

Example (19)
\n
$$
\int \frac{(3x-1)^2}{3x} dx
$$
\nBy using algebra :
$$
\int \frac{9x^2 - 6x + 1}{3x} dx = \int \frac{9x^2 - 6x}{3x} dx + \frac{1}{3x} dx = \int 3x - 2 + \frac{1}{3} \left(\frac{1}{x}\right) dx
$$
\n
$$
= \frac{3}{2}x^2 - 2x + \frac{1}{3}ln|x| + c
$$
\nExample (20)
\n
$$
\int \frac{6x^2 + 12}{x^3 + 6x + 1} dx
$$
\n
$$
\int \frac{x^2 + 2}{x^3 + 6x + 1} dx = \frac{6}{3} \int \frac{3x^2 + 6}{x^3 + 6x + 1} dx = 2ln|x^3 + 6x + 1| + c
$$
\nExample (21)
\n
$$
\int \frac{2e^x}{e^x + 1} dx
$$
\n
$$
\frac{\text{Answer}}{2} \int \frac{e^x}{e^x + 1} dx = 2ln|e^x + 1| + c
$$
\n
$$
\int \frac{e^x}{e^x + 1} dx = 2ln|e^x + 1| + c
$$

Example (22)

 dx $\int \frac{dx}{4x-1}$

Answer

$$
\frac{1}{4}\int \frac{4}{4x-1} dx = \frac{1}{4}ln|4x-1| + c
$$

Example (23)

 $\frac{\sec x \tan x}{\cos x} dx$ $\int \frac{\sec x 1}{\sec x - 1}$

Answer

$$
\int \frac{\sec x \tan x}{\sec x - 1} = \ln |\sec x - 1| + c
$$

$$
\int \frac{f'(x)}{f(x)}
$$

Example (24)

2 1 $\int \frac{1}{x \ln x^2} dx$

Answer

$$
\int \frac{1}{2x \ln x} dx = \frac{1}{2} \int \frac{1}{x \ln x} dx = \frac{1}{2} \int \frac{\frac{1}{x} e^{-\int f'(x)}}{\ln x} dx = \frac{1}{2} \ln |\ln x| + c
$$

Example (25)

Answer

$$
\int \frac{4}{x \ln 3x} dx
$$

$$
4 \int \frac{\frac{1}{x}}{\ln 3x} dx = 4 \ln |\ln 3x| + c
$$

$$
\int f(x)
$$

Techniques of integration

Introduction

Integration is more chanllenging than differentiation, in finding the derivative of a function it is obvious which differential formula should apply. But it may not be obvious which technique we should use to integrate a given function . Integration is more chanllenging than differentiation, in finding the derivative of a funct
it is obvious which differential formula should apply. But it may not be obvious which
technique we should use to integrate a give

 I *product or quotie nt h differential formula should apply. But it may not be obvic*
*cold use to integrate a given function .
I the integral of the product or quotient between two fun*
Can the integral be solved directly by the previous me

The idea behind using the substitution method is to replace a relatively complicated integral ind using the subst.

one, this is accomp

(1) Assume that a ma using the substite

one, this is accomponent

(1) Assume that a component of

(2) Integrate the function *by a simpler one, this is accomplished by changing the original variable x into u or z or*

- (1) Assume that a complicated function $=u$ *Steps*
	-
- one, this is accomplished by changing the original variation (1) Assume that a complicated function = u
(2) Integrate the function by any of the previous rules .
(3) If your function becomes more complicated than the
fu (2) Integrate the fu (3) If your function
function \Rightarrow "
(4) Return the function *3* Integrate the function by any of the previous rules 1 Integrate the function by any of the previous rules 1 *A s s s f f your function becomes more complicated than the original* (2) Integrate the function by any of the previor(3) If your function becomes more complicate
function \Rightarrow " change your assumption " *4 fu*
4 R
4 R function \Rightarrow "change your assumption" (3) If your function
function \Rightarrow "
(4) Return the func
(5) Simplify your is
	- *eturn the function back to its original variable . function* \Rightarrow "*change your assumption*"
 4) *Return the function back to its original vari*
 5) *Simplify your integral to the simplest form*.
	-

Examples

Example (1)

Ask your self

 algebra or rules

() *1 Integration by*

() *2 Integration by*

 Substitution

 $(e^{x}+1)^{2} dx$

Example (1)

\nFind:
$$
\int 2e^{x}(e^{x} + 1)^{2} dx
$$

\nStep (1): Let $u = e^{x} + 1 \Rightarrow du = e^{x} dx \rightarrow dx = \frac{du}{e^{x}}$

\n $\therefore I = \int \frac{2e^{x}u^{2}}{e^{x}} du = \int 2u^{2} du$

\nAs you see the integration here is more simpler than the original or

Step (1): Let
$$
u = e^x + 1 \implies du = e^x \, dx \to d
$$
.
\n
$$
\therefore I = \int \frac{2 \, e^x u^2}{e^x} \, du = \int 2u^2 \, du
$$
\nAs you see the integration here is more simpler that

n the original erral
 one n

Step (1): Let
$$
u = e^x + 1 \implies du = e^x dx \rightarrow dx = \frac{e^x}{e^x}
$$

\n
$$
\therefore I = \int \frac{2e^x u^2}{e^x} du = \int 2u^2 du
$$
\nAs you see the integration here is more simpler than the original one\nStep (2): $I = 2 \times \frac{u^3}{3} + c = \frac{2}{3}u^3 + c \implies$ Step (3): $I = \frac{2}{3}(e^x + 1)^3 + c$