Example (2)

*Example (2)
<i>Determine the increasing and decreasing intervals of the function : f (x)* = $x^3 - 3x^2 - 9x + 1$
Answer Answer mine the increasing
(x) is differentiable
 $f'(x) = 2x^2 - 6x - 9$

Determine the increasing and decreasing intervals of the function : $f(x) = x^3$
 \therefore $f(x)$ is differentiable and continous on R

(1) $f'(x) = 3x^2 - 6x - 9 = 3(x-3)(x+1) \implies$ (2) for $f'(x) = 0$ *1 f (x)* is differentiable and continuous on R
 1) $f'(x) = 3x^2 - 6x - 9 = 3(x-3)(x+1) \implies$ (2) for $f'(x) = 0$ *:* $f(x)$ *is differentiable and continous on R*
(1) $f'(x) = 3x^2 - 6x - 9 = 3(x - 3)(x + 1) \implies$ (2) for f'(
 \therefore $\boxed{x = 3}$ *and* $\boxed{x = -1}$ *are the critical points* differentiable and continuous on R
= $3x^2 - 6x - 9 = 3(x-3)(x+1) \Rightarrow$ (2) for $f'(x) =$: $f(x)$ is differentiable and continous on R
(1) $f'(x) = 3x^2 - 6x - 9 = 3(x-3)(x+1) \implies$ (2) for f'(
 \therefore $\overline{x=3}$ and $\overline{x=-1}$ are the critical points

Example (6)

Example (6)
Determine the increasing and decreasing intervals of the function : f (x) = $|2x-3|-x+2|$
Answer

Example (6)
\nDetermine the increasing and decreasing intervals of the function :
$$
f(x) = |2x-3| - 3
$$

\n $f(x) =\begin{cases} 2x-3-x+2, & x \ge \frac{3}{2} \\ 3-2x-x+2, & x < \frac{3}{2} \end{cases} \Rightarrow f(x) =\begin{cases} x-1, & x \ge \frac{3}{2} \\ 5-3x, & x < \frac{3}{2} \end{cases}$
\n $\therefore f(x)$ is continuous at $x = \frac{3}{2}$ as $f(3) = f(3^-) = f(3^+) = \frac{1}{2}$
\nAnd $\therefore f(x)$ is a double function, we have to get the critical points from:
\n(i) Check if : $f'(\frac{3^+}{2}) \ne f'(\frac{3^-}{2})$ by using differentiability (ii) $f'(x) = 0$

$$
\begin{vmatrix}\n3-2x-x+2 & x < \frac{1}{2} & |5-3x & x < \frac{1}{2} \\
\therefore f(x) \text{ is continuous at } x = \frac{3}{2} \text{ as } f(3) = f(3) = f(3^{\circ}) = \frac{1}{2} \\
\text{And } \therefore f(x) \text{ is a double function, we have to get the critical points from:}
$$
\n
$$
(i) \text{Check if: } f'\left(\frac{3}{2}\right) \neq f'\left(\frac{3}{2}\right) \text{ by using differentiability} \qquad (ii) f'(x) = 0
$$
\n
$$
(i) \text{For } f'\left(\frac{3}{2}\right) : \lim_{h \to 0^+} \frac{f\left(\frac{3}{2}+h\right)-f\left(\frac{3}{2}\right)}{h} \Rightarrow \lim_{h \to 0^+} \frac{f\left(\frac{3}{2}+h\right)-1-\left(\frac{3}{2}-1\right)}{h}
$$
\n
$$
\therefore \lim_{h \to 0^+} \frac{3+h-1-\frac{3}{2}+1}{h} \Rightarrow \lim_{h \to 0^+} \frac{h}{h} \Rightarrow \lim_{h \to 0^+} I = I
$$
\n
$$
\text{For } f'\left(\frac{3}{2}\right) : \lim_{h \to 0^+} \frac{f\left(\frac{3}{2}+h\right)-f\left(\frac{3}{2}\right)}{h} \Rightarrow \lim_{h \to 0^+} \frac{1}{h} \Rightarrow \lim_{h \to 0^+} J = 1
$$
\n
$$
\therefore \lim_{h \to 0^+} \frac{5-\frac{9}{2}-3h-5+\frac{9}{2}}{h} \Rightarrow \lim_{h \to 0^+} \frac{-3h}{h} \Rightarrow \lim_{h \to 0^+} J = -3
$$
\n
$$
\text{So: } \therefore f'\left(\frac{3}{2}\right) \neq f'\left(\frac{3}{2}\right) \text{ doesn't exist} \Rightarrow \left[\frac{x-3}{2}\right] \text{ is a critical point}
$$
\n
$$
(ii) \therefore f'(x) = \begin{cases} 1 & x \geq \frac{3}{2} \\ -3 & x < \frac{3}{2} \end{cases}, \text{ so we can't find } f'(x) = 0 \Rightarrow \left[\frac{x-3}{2}\right] \text
$$

Example (7)

Determine the increasing and decreasing intervals of the function : f (x) = $x|x-2|$

etermine the increasing and decreasin
 $(x) =\begin{cases} x(x-2) & x \ge 2 \\ -x(x-2) & x < 2 \end{cases} \Rightarrow f(x)$ **Example (7)**
the increasing and decreasing intervals of the funct
 $(x-2)$ $x \ge 2$
 $(x-2)$ $x < 2$ \Rightarrow $f(x) = \begin{cases} x^2 - 2x, & x \ge 2 \\ 2x - x^2, & x < 2 \end{cases}$
continuous at $x = 2$ as $f(2) = f(2^-) = f(2^+) = 0$ **Example (7)**

ine the increasing and decreasing intervals of the function : $f(x) = \frac{Answer}{2}$
 $\int x(x-2)$ $x \ge 2$
 $\Rightarrow f(x) = \begin{cases} x^2 - 2x, & x \ge 2 \\ 2x - x^2, & x < 2 \end{cases}$ *2 x x 2 x 2 x 2x , x 2* Example (7)

ermine the increasing and decreasing intervals of the function : $f(x) = x|x-2|$
 $\Rightarrow f(x) =\begin{cases} x(x-2) & x \ge 2 \\ -x(x-2) & x < 2 \end{cases}$ $\Rightarrow f(x) =\begin{cases} x^2 - 2x, & x \ge 2 \\ 2x - x^2, & x < 2 \end{cases}$

(x) is continous at $x = 2$ as $f(2) = f(2^-$ **Example** (7)

mine the increasing and decreasing intervals of the fun
 $=\begin{cases} x(x-2) & x \ge 2 \\ -x(x-2) & x < 2 \end{cases}$ \Rightarrow $f(x) = \begin{cases} x^2 - 2x, & x \ge 2 \\ 2x - x^2, & x < 2 \end{cases}$ Determine the increasing and decreasin
 $f(x) = \begin{cases} x(x-2) & x \ge 2 \\ -x(x-2) & x < 2 \end{cases} \Rightarrow f(x)$ ine the increasing and decreasing intervals of the function : $f(x) = \frac{Answer}{2x(x-2)}$
 $\Rightarrow f(x) = \begin{cases} x^2 - 2x, & x \ge 2 \\ 2x - x^2, & x < 2 \end{cases}$ *a x* (*x* - 2) $x \ge 2$
 $-x(x-2)$ $x \ge 2$
 $-x(x-2)$ $x < 2$
 $\Rightarrow f(x) = \begin{cases} x^2 - 2x, & x \ge 2 \\ 2x - x^2, & x < 2 \end{cases}$ *Answer* $\begin{aligned}\n\therefore \\
\Rightarrow \quad f(x) = \begin{cases}\nx^2 - 2x, & x \ge 2 \\
2x - x^2, & x < 2\n\end{cases} \\
\Rightarrow \quad \text{as } f(2) = f(2^-) = f(2^+) = 0\n\end{aligned}$ $f(x) = \begin{cases} x(x-2) & x \ge 2 \\ -x(x-2) & x < 2 \end{cases} \Rightarrow f(x) = \begin{cases} x^2 - 2x, & x \ge 2 \\ 2x - x^2, & x < 2 \end{cases}$
 $\therefore f(x)$ is continous at $x = 2$ as $f(2) = f(2^-) = f(2^+) =$
 $\therefore f(x)$ is a double function, we have to get the critical po

(i) Check if \therefore $f(x) =\begin{cases} x(x-2) & x \ge 2 \\ -x(x-2) & x < 2 \end{cases} \implies f(x) =\begin{cases} x^2 - 2x, & x \ge 2 \\ 2x - x^2, & x < 2 \end{cases}$
 $f(x)$ is continous at $x = 2$ as $f(2) = f(2^-) = f(2^+) = 0$ *2* $f(x) =\begin{cases} x(x-2) & x \neq 2 \\ -x(x-2) & x < 2 \end{cases}$ \Rightarrow $f(x) =\begin{cases} x - 2x \\ 2x - x^2 \end{cases}$, $x < 2$
 $f(x)$ is continuos at $x = 2$ as $f(2) = f(2^-) = f(2^+) = 0$
 $f(x)$ is a double function, we have to get the critical points from: \Rightarrow $f(x) = \begin{cases} x^2 - 2x^3, & x \le 2 \\ 2x - x^2, & x < 2 \end{cases}$
 $x = 2$ as $f(2) = f(2^-) = f(2^+) = 0$
 \Rightarrow $f'(2^-)$ by using defferentiability \Rightarrow $f'(2^-) = 0$
 \Rightarrow $f(2) = 2(2+h) - 2 - [2(2)-2]$ $\begin{array}{lll} & (-x(x-2) & x<2 & (2x-x^2), x<2 \end{array}$
 $\therefore f(x) \text{ is continuous at } x=2 \text{ as } f(2)=f(2^-)=f(2^+) = 0$
 $\therefore f(x) \text{ is a double function, we have to get the critical points from:}$

(i) Check if $\therefore f'(2^+) \neq f'(2^-)$ by using defferentiability (ii) $f'(x)=0$

(i) For $f'(2^+) \therefore \lim_{h \to 0^+} \frac{f(2+h)-f(2$ *i* $f(x)$ is continous at $x = 2$ as $f(2) = f(2^-) = f(2^-)$
i $f(x)$ is a double function, we have to get the critic *i*) Check if $f'(2^+) \neq f'(2^-)$ by using defferentia $\begin{aligned} (2^+) &= 0 \\ \text{cal points from:} \\ \text{bility} & \text{(ii) } f'(x) = 0 \\ \text{d} \end{aligned}$ $\frac{1}{2}$ = $\frac{1}{2}$ = $\frac{1}{2}$ + $\frac{1}{2}$ + $\frac{1}{2}$ + $\frac{1}{2}$ + $\frac{1}{2}$ + $\frac{1}{2}$ by usin on, we have to get the critical points from:
 $f'(2^-)$ by using defferentiability $(i) f'(x) = 0$
 $\frac{f'(2^-)}{h}$ \Rightarrow $\lim_{h \to 0^+} \frac{2(2+h)-2-[2(2)-2]}{h}$ *f f* $f'(2^{-})$ *by using defferentiability* (*ii* $f(2+h)-f(2) \Rightarrow \lim_{h \to 0^+} \frac{2(2+h)-2-[2(2)-2]}{h}$ *i* f(x) is a double function, we have to get th
 i) Check if : $f'(2^+) \neq f'(2^-)$ by using deffering the *i*) For $f'(2^+)$: $\lim_{h \to 0^+} \frac{f(2+h)-f(2)}{h} \Rightarrow \lim_{h \to 0^+}$ we have to get the critical point

2⁻) by using defferentiability
 $\frac{h}{h}$ \Rightarrow $\lim_{h\to 0^+} \frac{2(2+h)-2}{h}$ Check if : $f'(2^+) \neq f'(2^-)$ by using defferentiability (ii)

For $f'(2^+)$: $\lim_{h \to 0^+} \frac{f(2+h)-f(2)}{h} \Rightarrow \lim_{h \to 0^+} \frac{2(2+h)-2-[2(2)-2]}{h}$
 $\therefore \lim_{h \to 0^+} \frac{4+2h-2-4+2}{h} \Rightarrow \lim_{h \to 0^+} \frac{2h}{h} \Rightarrow \lim_{h \to 0^+} 2=2$ $(ii) f'(x) = 0$ (*i*) Check if $f: f'(2^+) \neq f'(2^-)$ by using defferentiability

(*i*) For $f'(2^+)$: $\lim_{h \to 0^+} \frac{f(2+h)-f(2)}{h} \Rightarrow \lim_{h \to 0^+} \frac{2(2+h)-f(2)}{h}$
 $\therefore \lim_{h \to 0^+} \frac{4+2h-2-4+2}{h} \Rightarrow \lim_{h \to 0^+} \frac{2h}{h} \Rightarrow \lim_{h \to 0^+} 2 = 2$ $y : J(2) \neq J(2)$ by using aey perentiability $(u) : (u) \neq 0$
 $(2^+) : \lim_{h \to 0^+} \frac{f(2+h)-f(2)}{h} \Rightarrow \lim_{h \to 0^+} \frac{2(2+h)-2-[2(2)-2]}{h}$
 $\frac{4+2h-2-4+2}{h} \Rightarrow \lim_{h \to 0^+} \frac{2h}{h} \Rightarrow \lim_{h \to 0^+} 2 = 2$
 $(2^-) : \lim_{h \to 0^+} \frac{f(2+h)-f(2)}{h} \Rightarrow \lim$ $\lim_{h \to 0^+} \frac{f(2+h)-f(2)}{h} \Rightarrow \lim_{h \to 0^+} \frac{2(2+h)}{h}$ $\lim_{h\to 0^+} \frac{f(2+h)-f(2)}{h} \Rightarrow \lim_{h\to 0}$ $\frac{f(2+h)-f(2)}{h}$ \Rightarrow $\lim_{h\to 0^+} \frac{2(2+h)-f(2)}{h}$ $\frac{2h}{h}$ \Rightarrow $\lim_{h\to 0^+} \frac{2h}{h}$ \Rightarrow $\lim_{h\to 0^+} 2$
 $f(2+h)-f(2)$ \Rightarrow 2-2 + $\frac{h}{h} \Rightarrow \lim_{h \to 0^+} \frac{h}{h}$
 $\frac{h}{h} \Rightarrow \lim_{h \to 0^+} 2 = 2$
 $\frac{h}{h} \Rightarrow \lim_{h \to 0^+} 2 = 2$
 $\frac{h}{h} \Rightarrow \lim_{h \to 0^+} 2 - 2(x+2) - 2(2)$ $\frac{2}{h}$ *h* $\frac{2h}{h\rightarrow 0^{+}}$ *im* $\frac{2}{h}$ *h* $\frac{2}{h\rightarrow 0^{+}}$ *h* $\frac{2}{h\rightarrow 0^{+}}$ *h* $\frac{2}{h\rightarrow 0^{+}}$ *z* = 2
 $\frac{f(2+h)-f(2)}{h}$ \Rightarrow $\lim_{h\rightarrow 0^{+}}$ $\frac{2-2(x+2)-[2-2(2+h)-f(2)]}{h}$ $\lim_{h\to 0^+}\frac{4+2h-2-4+2}{h} \Rightarrow \lim_{h\to 0^+}\frac{2h}{h} \Rightarrow \lim_{h\to 0}$ \therefore $\lim_{h\to 0^+} \frac{4+2h-2-4+2}{h} \Rightarrow \lim_{h\to 0^+} \frac{2h}{h} \Rightarrow$
 $\text{For } f' (2^-): \lim_{h\to 0^+} \frac{f(2+h)-f(2)}{h} \Rightarrow \lim_{h\to 0^+}$ $\Rightarrow \lim_{h \to 0^+} \frac{2h}{h} \Rightarrow \lim_{h \to 0^+} 2 = 2$
 $\lim_{h \to 0^+} \frac{h}{h} \Rightarrow \lim_{h \to 0^+} \frac{2 - 2(x + 2) - 2}{h}$ ∴ $\lim_{h\to 0^+} \frac{f(2+h)-f(2)}{h}$ ⇒ $\lim_{h\to 0^+} \frac{2h}{h}$ ⇒ $\lim_{h\to 0^+}$

For $f'(2^-)$: $\lim_{h\to 0^+} \frac{f(2+h)-f(2)}{h}$ ⇒ $\lim_{h\to 0^+} \frac{2-1}{h}$

∴ $\lim_{h\to 0^-} \frac{2-4-2h-2+4}{h}$ ⇒ $\lim_{h\to 0^-} \frac{-2h}{h}$ ⇒ $\lim_{h\to 0^-}$

So : $f'(2^$ $\therefore \lim_{h\to 0^+} \frac{h}{h}$
 For $f'(2^-)$: $\lim_{h\to 0^+} \frac{f(2+h)}{h}$
 $\therefore \lim_{h\to 0^-} \frac{2-4-2h-2+4}{h}$ \rightarrow 0⁺ h h h h h h h h h *h* \rightarrow *h* h $h \rightarrow 0^{+}$
 $\frac{c^{n}(2)}{h}$ \rightarrow $\lim_{h \rightarrow 0^{+}} \frac{2 - 2(x + 2) - [2 - 2(2 + 2)]}{h}$
 $\lim_{h \rightarrow 0^{-}} \frac{-2h}{h}$ \rightarrow $\lim_{h \rightarrow 0^{-}} -2 = -2$ $\lim_{h\to 0^+} \frac{f(2+h)-f(2)}{h} \Rightarrow \lim_{h\to 0^+} \frac{2-2h}{h}$ $\lim_{h\to 0^+} \frac{f(2+h)-f(2)}{h} \Rightarrow \lim_{h\to 0^+}$ $\frac{f(2+h)-f(2)}{h} \Rightarrow \lim_{h\to 0^+} \frac{2-2}{h}$ − For $f'(2^-)$: $\lim_{h\to 0^+} \frac{3h}{h} \Rightarrow \lim_{h\to 0^+} \frac{2h}{h}$
 $\therefore \lim_{h\to 0^-} \frac{2-4-2h-2+4}{h} \Rightarrow \lim_{h\to 0^-} \frac{-2h}{h} \Rightarrow \lim_{h\to 0^-} -2 = -2$

So $\therefore f'(2^+) \neq f'(2^-)$ doesn't exist $\Rightarrow \boxed{x=2}$ is the a critical point *h*→0⁺
 $\lim_{h\to 0^-}$ $\frac{-2h}{h}$ \Rightarrow $\lim_{h\to 0^-}$ - 2 = -2 *h*→0 *h*
 h→0

 h ∴ $\lim_{h\to 0^{-}} \frac{2-4-2h-2+4}{h} \Rightarrow \lim_{h\to 0^{-}}$
So : $f'(2^+) \neq f'(2^-)$ doesn't
(ii) : $f'(x) = \begin{cases} 2x-2 & x \ge 2 \\ 2-2x & x < 2 \end{cases}$ $\frac{4-2h-2+4}{h} \Rightarrow \lim_{h\to 0^-} \frac{-2h}{h} \Rightarrow$
 \Rightarrow
 \Rightarrow $\Rightarrow f'(2^-) \text{ doesn't exist } \Rightarrow$
 \Rightarrow
 \Rightarrow $\begin{cases} 2x-2 & x \ge 2 \\ 2-2x & x < 2 \end{cases}$ $\frac{-2h-2+4}{h} \Rightarrow$
 $\neq f'(2^-)$ does:
 $2x-2$ $x \ge 2$
 $2-2x$ $x < 2$ $\therefore \lim_{h \to 0^{-}} \frac{1}{h}$
So $\therefore f'$
ii) $\therefore f'(x)$ h $h \rightarrow 0^-$ h
 $\neq f'(2^-)$ doesn't exist ⇒ $\begin{cases} 2x-2 & x \ge 2 \\ 2-2x & x < 2 \end{cases}$ *h*
 $\neq f'(2^-)$ does:
 $2x - 2$ $x \ge 2$
 $2 - 2x$ $x < 2$ *So* $f'(2^x) \neq f'(2^x)$ doesn't exist $\Rightarrow |x| \geq 2$ is the a critical point

(ii) $f'(x) = \begin{cases} 2x-2 & x \geq 2 \\ 2-2x & x < 2 \end{cases}$

<u>For $x \geq 2$ </u> : $2x-2=0 \Rightarrow |x=1|$ is a critical point but $x \geq 2$, $\boxed{\text{so } x=1 \text{ is refused}}$ $f'(x) =\begin{cases} 2x-2 & x \ge 2 \\ 2-2x & x < 2 \end{cases}$
 $x \ge 2$: $2x-2=0 \Rightarrow \boxed{x=1}$ is a critical point $x < 2$: $2-2x=0 \Rightarrow \boxed{x=1}$ is a critical point $=\{$ *For* $x < 2$: $2-2x=0$ \Rightarrow $\boxed{x=1}$ *is a critical point* $\begin{array}{|l|c|c|c|c|}\n\hline\n x = 0 & x = 1 & x = 1.5 & x = 2 & x = 10 \\
\hline\n f'(0) = +ve & f'(1.5) = -ve & f'(10) = +ve\n\end{array}$ *Increasing Decreasing Increasing ++++++++++++++ 0 - - - - - - - - - - - - - - - - - - 0 ++++++++++++++* $++++$
 (x) is ine (z) is ded
(x) is ded $\begin{array}{c} \text{+++++}\\ \text{(x)} \text{ is } \text{in} \end{array}$ $f(x)$ is increasing on $f(x)$ is decreasing over $f(x)$ is increasing over $\mathbf{\hat{I}}$ x) is increasi
] -∞, 1['s decreasing
] -1, 2 [is increasing $\left] 2,\infty \right[$ *-1, 2* $\frac{1}{2}$, ∞ $]-\infty, 1[$ $f(x)$ has a local maximum $f(x)$ has a local minimum $f(x)$ has a loce
at $x = 1$ $f(x)$ has a loce
at $x = 2$

Example (8)

Find the intervals where the following function is increasing or decreasing where :

Find the intervals where the following function is increasing or decreasing where:
\n
$$
f(x) = \sin x(1 + \cos x)
$$
 in $\left[0, \frac{\pi}{2}\right]$
\n $f'(x) = \sin x(-\sin x) + (1 + \cos x)\cos x \Rightarrow f'(x) = -\sin^2 x + \cos x + \cos^2 x$
\nRemember that: $\sin^2 x = 1 - \cos^2 x \Rightarrow f'(x) = -(1 - \cos^2 x) + \cos x + \cos^2 x$
\n $\therefore f'(x) = 2\cos^2 x + \cos x - 1 \Rightarrow \text{ for } f'(x) = 0 \Rightarrow (2\cos x - 1)(\cos x + 1) = 0$
\n $\therefore \cos x = \frac{1}{2}$ and $\cos x = -1$
\nThen $\left[x = \cos^{-1} \frac{1}{2} = 60^\circ\right]$
\n $\text{Then } \left[x = \cos^{-1} \frac{1}{2} = 60^\circ\right]$
\n \therefore The only critical point is $\left[x = 60^\circ\right]$
\n $\left[\frac{x = 30^\circ}{x} - \frac{300^\circ}{x} - \frac{300^\circ}{x} - \frac{300^\circ}{x} + \frac{300^\circ}{x} - \frac{300^\circ}{x} + \frac{$

<u>Then $x = 60^\circ$ *is a critical point as the sign of curvature changed*</u>

Example (9)
Find the intervals where the following function is increasing or decreasing where :

 $f(x) = 2\sin x \cos x$ in $[0,\pi]$

Answer

Special cases of increasing and decreasing functions problems "Fractional functions"

Remember that In any fractional function, critical points occurs if :

- Remember that
 In any fr

(1) $f'(x)$ exists and equals

(2) $f'(x)$ doesn't exist." () () *1 f ' x exists and equals to zero ." Numerator 0"* =
- *2 f'* (*x*) exists and equals to zero ." Numerator = 0"
 2 f' (*x*) doesn't exist " undefined when denomenator = 0"
- * So, the first step to solve the fractional problems is to find the domain *(1)* $f'(x)$ exists and equals to zero ." Numerator = 0"
 (2) $f'(x)$ doesn't exist " undefined when denomenator * So, the first step to solve the fractional problems is to
- (2) $f'(x)$ doesn't exist " undefined when denomenator =
* So, the first step to solve the fractional problems is to $\underline{\mathbf{f}^*}$
* Then determine the increase and decrease intervals over ^{*} Then determine the increase and decrease intervals over the domain

Example (11)

Example (11)
Determine the increasing and decreasing intervals of the function : $f(x) = \frac{1}{3}x - \sqrt[3]{x}$ *
Answer* $=\frac{1}{3}x-\sqrt[3]{x}$

Answer

Case (2) : Local maximum and local minimum
 We can get local maximum or local minimum by two ways , either :

(1) By using 1st derivative test "as we have done before in the previous case" *" Classification"* **Case (2) : Local maximum and local minimum**
 1 Classification"
 Ne can get local maximum or local minimum by two ways , either :

(1) By using 1st derivative test "as we have done before in the previous case"
 Note

We can get local maximum or local minimum by two way

(1) By using 1st derivative test "as we have done before in
 Note : we use this method if the function was linear

(2) By using 2nd derivative test

Note: we use this method if the function was linear " *its power is one* "

Ne can get local maximum or lo

(1) *By using 1st derivative test* "
 Note : we use this method if

(2) *By using 2nd derivative test*
 Note : we use this method if

 Note : we use this method if he asked me to find local maximum or minimum directly. (2) By using 2^{nd} derivative test

Note : we use this method if he asked me to find local maximum or minimum directly.

(1) Find by differentiation the first derivative of the given function $f'(x)$.

(2) Find the critic

Steps

- Note : we use this method if he asked me to find local r

(1) Find by differentiation the first derivative of the given fur

(2) Find the critical points using $f'(x) = 0$, you will get the va

(3) Get the second derivative (1) Find by differentiation
(2) Find the critical points
(3) Get the second derivat
(4) Substitute the critical (1) Find by differentiation
(2) Find the critical points
(3) Get the second derivation
(4) Substitute the critical
(5) (i) If the result is + ve *' 1 Find by differentiation the first derivative of the given function f'*(*x*).
 2 Find the critical points using $f'(x) = 0$, you will get the value of *x* for *t* 1) Find by differentiation the
2) Find the critical points usi
3) Get the second derivative .
4) Substitute the critical poir
- *' 2* **2** *2 Find by differentiation the first derivative of the given function f'(x).*
2) Find the critical points using f'(x)=0, you will get the value of x for this critical point.
3) Get the second derivative . (1) Find by differentiation the first derivative of the given function

(2) Find the critical points using $f'(x) = 0$, you will get the value o

(3) Get the second derivative .

(4) Substitute the critical points if exists

Find the critical points use
Get the second derivative.
Substitute the critical poin
(i) If the result is - ve
(ii) If the result is - ve Get the second derivative .

Substitute the critical poin

(i) If the result is + ve

(ii) If the result is - ve

(iii) If the result is zero

Substitute the critical poin *f* is the second derivative .
 5) Get the second derivative .

4) Substitute the critical points if exists in the second de
 5) (i) If the result is + ve …… Then it is local minimum.

(ii) If the result is - ve …… *(3)* Get the second derioutioe .

(4) Substitute the critical points if exists in the second der $(5)(i)$ If the result is + ve …… Then it is local minimum.
 (ii) If the result is - ve …… Then it is local maximum.
 (iii

(iii) If the result is zero …… Then it is Neither local minimum Nor local maximum *(ii)* If the result is - ve Then it is local maximum.
(iii) If the result is zero Then it is Neither local minimum Nor
(6) Substitute the critical point is the original function to get the point

(4) Substitute the critical

(5) (i) If the result is + ve

(ii) If the result is - ve

(iii) If the result is zer

(6) Substitute the critical

Example (1) (*iii*) *If the result is zero* Then it is Neither local minimum Nor local maximum
6) Substitute the critical point is the original function to get the point
Example (1)
Find the local maximum and the local mini *Answer f* ind the local maximent
f (*x*) is continous on *R*

Substitute the critical point is the original function to get the point
 $Example (1)$

ind the local maximum and the local minimum of $f(x) = x^4 - 18$
 (x) is continous on R
 $f'(x) = 4x^3 - 36x = 4x(x^2 - 9) = 4x(x - 3)(x + 3)$

or $f'(x) =$ *he local maximum and*
 s continous on R
 $= 4x^3 - 36x = 4x(x^2 - 9)$
 $(x) = 0 \Rightarrow x = 0$ a
 $(x) = 12x^2 - 36$ is continous on R
= $4x^3 - 36x = 4x(x^2 - 9)$
 $(x)=0 \Rightarrow \boxed{x=0}$ a
 $(x)=12x^2 - 36$
 $(a) = -36 < 0 \Rightarrow \boxed{3}$ $=4x^3-36x=4x(x^2-9)$
 $'(x)=0 \Rightarrow \boxed{x=0}$
 $'(x)=12x^2-36$
 $(0)=-36 < 0 \Rightarrow \boxed{\therefore a}$ *ntinous on* R
 $x^3 - 36x = 4x(x^2)$ \therefore $f''(x) = 12x^2 - 36$ *f f (x) is continous on R*
 f $'(x) = 4x^3 - 36x = 4x(x^2 - 9) = 4x(x - 3)(x + 3)$ *f* (*x*) is continous on *R*
 $f'(x) = 4x^3 - 36x = 4x(x^2 - 9) = 4x(x - 3)(x + 3)$
 For f' (*x*) = 0 \Rightarrow $\boxed{x = 0}$ and $\boxed{x = 3}$ and $\boxed{x = -3}$ *f'*(*x*) = $4x^3 - 36x = 4x(x^2 - 9) = 4$
 For f'(*x*) = 0 \Rightarrow $x = 0$ and
 $\therefore f''(x) = 12x^2 - 36$ *for* $f'(x) = 0 \Rightarrow \boxed{x = 0}$ and $\boxed{x = 3}$ and $\boxed{x = -3}$
 $\therefore f''(x) = 12x^2 - 36$
 $\therefore f''(0) = -36 < 0 \Rightarrow \boxed{\therefore at x = 0} \Rightarrow the function has a local maximum$ ** After substituting in the o* $\frac{um}{(0,0)}$ $r(x) = 12x^2 - 36$
 $(0) = -36 < 0 \Rightarrow \boxed{\therefore at x = 0} \rightarrow the function has a local maximum$
 r substituting in the original function \Rightarrow the local maximum point is $\boxed{(0,0)}$
 $(3) = 72 > 0 \Rightarrow \boxed{\therefore at x = 3} \rightarrow the function has a local minimum$
 r substituting in the original function \Rightarrow t $\begin{array}{c}\n(m \\
 (0,0) \\
 (3,-81)\n\end{array}$ After substituting in the origit
 $f''(3)=72>0$ \Rightarrow $\boxed{\therefore}$ and

After substituting in the origit
 $f''(-3)=72>0$ \Rightarrow $\boxed{\therefore}$ and $f''(0) = -36 < 0 \Rightarrow$ \therefore at $x = 0$ \Rightarrow the function has a local maximum
 $f(r)(3) = 72 > 0 \Rightarrow$ \therefore at $x = 3$ \Rightarrow the function has a local minimum * *After substituting in the original function* \Rightarrow *the local maximum point is* $(0,0)$
 $f''(3)=72>0$ \Rightarrow $\boxed{\therefore at x=3}$ \rightarrow *the function has a local minimum*

* *After substituting in the original function* \Rightarrow *the l* \Rightarrow = um
 $(3, -81)$

um
 $(-3, -81)$ *o* \Rightarrow $\boxed{\therefore at x = 3}$ \rightarrow *the function has a local minimum*
ng in the original function \Rightarrow *the local maximum point is* $\boxed{(3, 0)}$ \Rightarrow $\boxed{\therefore at x = -3}$ \rightarrow *the function has a local minimum* * *After substituting in the original function* \Rightarrow *the local maximum point is* $(3, -81)$
 f "(-3)=72>0 \Rightarrow \therefore *at* $x = -3$ \rightarrow *the function has a local minimum*

* *After substituting in the original function* \Rightarrow \therefore at $x = 3$ \Rightarrow the function h
ting in the original function \Rightarrow the local m
 >0 \Rightarrow \therefore at $x = -3$ \Rightarrow the function h * After substituting in the original function \Rightarrow the local maximum point is $|($ -3, -81 $)|$

Example (2) **Example (2)**
Classify the function f where $f(x) = 3x^5 + 2x^3 - 1$ \hat{a} assify the function
 (x) is continous of

Answer

Classify the function
$$
f
$$
 where $f(x) = 3x^5 + 2x^3 - 1$

\n $f(x)$ is continuous on R

\n $\therefore f'(x) = 15x^4 + 6x^2 = 3x^2(5x^2 + 2)$, for $f'(x) = 0$

\n $\therefore 3x^2(5x^2 + 2) = 0 \implies x = 0$ or $x^2 = \frac{-2}{5}$ "refused"

\n $f''(x) = 60x^3 + 12x$

\n $f''(0) = 0 \implies$ Neither local maximum Nor minimum

Example (3)

2 2 $f''(0)=0$ \Rightarrow *Neither local miaximum Nor minimum*
Example (3)
Investigate the local maximum and the local minimum of $f(x) = \frac{1-x}{1+x}$ $\frac{l - x}{l + x}$ − = +

Calculus is $\Delta E = \int (X - \lambda)^2 dx$ *<i>Calculus Example (3)*
 Calculus is the total maximum and the forced minimum of $f(x) = \frac{1 - x^2}{1 + x^2}$
 Calculus is frection $\Rightarrow \therefore$ the domination of $f(x)$ is continuous on R for the cr *Answer* (0)=0 \Rightarrow Neither local miaximum Nor minimum
Example (3)
estigate the local maximum and the local minimum of $f(x) = \frac{1 - x^2}{1 + x^2}$
The problem is fraction \Rightarrow : the domain of $f(x)$ is R and $f(x)$ is continous on R **Example** (3)
the local minimum of $f(x) = \frac{1 - x^2}{1 + x^2}$
Answer
he domain of $f(x)$ is R and $f(x)$ is continous
 $\frac{(1 + x^2) - 2x(1 - x^2)}{(1 + x^2)^2} = \frac{-4x}{(1 + x^2)^2}$ Example (3)

local minimum of $f(x) = \frac{1-x^2}{1+x^2}$

Answer

lomain of $f(x)$ is R and $f(x)$ is continous on R
 $\frac{x^2}{1+x^2} - 2x(1-x^2) = \frac{-4x}{(1+x^2)^2}$
 $\Rightarrow 1 + 4x = 0 \Rightarrow x = 0$ The problem is fraction \Rightarrow \therefore the domain of $f(x)$ is R and

To get the critical points: $y' = \frac{-2x(1+x^2) - 2x(1-x^2)}{(1+x^2)^2} = \frac{1}{(1+x^2)^2}$

(1) For $f'(x) = 0 \Rightarrow \therefore \frac{-4x}{(1+x^2)^2} = 0 \Rightarrow \therefore -4x = 0 \Rightarrow \therefore \frac{1}{(1+x^2)^2} = 0 \Rightarrow \text{here$ To get the critical points: $y' = \frac{-2x(1+x^2) - 2x(1-x^2)}{(1+x^2)^2} = \frac{-4}{(1+x^2)}$

(1) For $f'(x) = 0 \implies \therefore \frac{-4x}{(1+x^2)^2} = 0 \implies \therefore -4x = 0 \implies \frac{\therefore x}{\therefore x}$

(2) $f'(x)$ is undefined when $(1+x^2)^2 = 0 \implies$ here x has no real Then the onl $\frac{Answer}{main of f(x)i}$ $2) - 2x(1-x^2)$ *2 2 -2x 1 x 2x 1 x -4x To get the critical points: y' 2 2 2 2 Example* (3)
 estigate the local maximum and the local minimum of $f(x) = \frac{1 - x^2}{1 + x^2}$
 Answer
 The problem is fraction \Rightarrow \therefore *the domain of* $f(x)$ *is R and* $f(x)$ *is continous on R Pomain of f* (x) *is R* and
 $\frac{x^2}{1+x^2}$ $\frac{-2x(1-x^2)}{1+x^2}$ $\frac{1}{x^2+x^2}$ $\frac{1}{x^2+x^2}$ *co get the critical points:* $y' = \frac{-2x(1+x^2) - 2x(1-x^2)}{(1+x^2)^2} = \frac{-4x}{(1+x^2)^2}$
 1) For f'(*x*)=0 \Rightarrow $\therefore \frac{-4x}{(1+x^2)^2} = 0 \Rightarrow \therefore -4x = 0 \Rightarrow \boxed{\therefore x = 0}$ $y' = -\frac{4y}{1 + x}$ *2) For* $f'(x)=0 \Rightarrow \therefore \frac{-4x}{(1+x^2)^2} = 0 \Rightarrow \therefore -4x$
2) f '(*x*) *is undefined when* $(1+x^2)^2 = 0 \Rightarrow$ *here*
Zhow the orthogonic moint is at $x = 0$ *.* the local minimum of $f(x) = \frac{1}{1+x}$

Answer

domain of $f(x)$ is R and $f(x) = \frac{1+x^2}{1+x^2} - 2x(1-x^2) = \frac{-4x}{(1-x^2)^2}$ Answer
 \Rightarrow :. the domain of $f(x)$ is R and $f(x)$ is $c = \frac{-2x(1+x^2)-2x(1-x^2)}{(1+x^2)^2} = \frac{-4x}{(1+x^2)^2}$ Answer

nain of $f(x)$ is R and $f(x)$ is continuon-
 $\left(-\frac{1}{x^2}\right)^2 - \left(\frac{1}{1+x^2}\right)^2$ cal points: $y' = \frac{-2x(1+x^2) - 2x(1-x^2)}{(1+x^2)^2} = \frac{-4x}{(1+x^2)^2}$
= $0 \implies \therefore \frac{-4x}{(1+x^2)^2} = 0 \implies \therefore -4x = 0 \implies \frac{\therefore x = 0}{(1+x^2)^2}$ $(1+x)$
 $(1+x)$
 $(1+x)$
 $(1+x)$
 $(1+x^2)^2 = 0 \Rightarrow \therefore -4x = 0 \Rightarrow \therefore \therefore$

i. $(1+x^2)^2 = 0 \Rightarrow$ here x has no re

only critical point is at $x = 0$
 $(1+x^2)^2 + 4x(1+x^2)4x$
 $(1+x^2)^4$ $\rightarrow \cdots$ $\overline{(1+x^2)}^2 = 0 \rightarrow \cdots -4x = 0$

ed when $(1+x^2)^2 = 0 \Rightarrow$ here x hot

itical point is at $x = 0$
 $\int^2 + 4x(1+x^2)4x$
 $(1+x^2)^4$
 \cdots $-4(1)^2$ int is at $x = 0$
 $\left(1 + x^2\right)4x$
 $\left(1\right)^2$
 $\left(1\right)^2 = -4 < 0 \Rightarrow$ the func $\frac{x(1+x^2)4x}{y^4}$
 $\frac{4(1)^2}{y^4} = -4 < 0 \Rightarrow$ the furties local maximum point is *2 2* \int_1^2 *2* \int_2^2 + 4x $\left(1 + x^2\right)$ (2) $f'(x)$ is undefined when $(1+x^2)^2$
 Then the only critical point is at x
 $\therefore y'' = \frac{-4(1+x^2)^2 + 4x(1+x^2)4x}{(1+x^2)^4}$ *2 4 x has no real solution* (1) For $f'(x)=0 \Rightarrow \therefore \frac{dx}{(1+x^2)^2}=0$

(2) $f'(x)$ is undefined when $(1+x^2)^2=0$

Then the only critical point is at $x=0$ $\begin{aligned} i\text{cal} \\ i\text{cal} \\ j^2 + 4 \\ \hline l + x \end{aligned}$ $\frac{4x(1)}{x^2}$
 $\frac{-4(1)}{(1)^4}$ ∴ $y'' = \frac{-4(1+x^2)^2 + 4x(1+x^2)4x}{(1+x^2)^4}$
 At $\boxed{x=0}$ \Rightarrow $y'' = \frac{-4(1)^2}{(1)^4} = -4 < 0$ \Rightarrow *the function has a local maximum at* $x = 0$ Then the only critical point is at $x = 0$ lefined when $(1 + x^2)^2 = 0 \Rightarrow h$
y critical point is at $x = 0$
 $+ x^2)^2 + 4x(1 + x^2)4x$ $f'(x)$ is undefined whe
Then the only critical p
 \therefore $y'' = \frac{-4(1+x^2)^2 + 4z}{(1+x^2)}$ $f(x) + 4x(1+x^2)^4$
 $\Rightarrow y'' = \frac{-4(1)^2}{(1)^4} = -4 < 0 \Rightarrow$ the function has a local maximum contract the local maximum point is $(0,1)$

Example (4)

Classify:
$$
y = x + \frac{4}{x - 1}
$$
, $x \ne 1$

Answer

Example (4)
 $+\frac{4}{x-1}$, $x \ne 1$

(x) is $R - \{1\} \Rightarrow f(x)$ is continuous on $R - \{1\}$

4 $(x-1)^2 - 4$ example 1 $x = x + \frac{4}{x-1}$, $x \ne 1$

(omain of $f(x)$ is $R - \{1\} \Rightarrow f(x)$ is co
 $(x) = 1 - \frac{4}{(x-1)^2} = \frac{(x-1)^2 - 4}{(x-1)^2}$ *Example (4)*
 $, x \neq 1$
 $-{1} \Rightarrow f(x)$ is continous or
 $(x-1)^2 - 4$
 $(x-1)^2$ $x \ne 1$
 ${1} \Rightarrow f(x)$ is continous on
 $(-1)^2 - 4$
 $(-1)^2$
 $(-1)^2 - 4$ The domain of $f(x)$ is $R - \{1\} \Rightarrow f(x)$ is continuous of
 $f'(x) = 1 - \frac{4}{(x-1)^2} = \frac{(x-1)^2 - 4}{(x-1)^2}$

(1) For $f'(x) = 0 \Rightarrow \frac{(x-1)^2 - 4}{(x-1)^2} = 0 \Rightarrow \therefore (x-1)^2$ Answer
 ${A \to f(x) \text{ is continuous on } R - \{1\}}$
 $\frac{(x-1)^2 - 4}{(x-1)^2}$
 $\frac{(-1)^2 - 4}{(x-1)^2} = 0 \implies (x-1)^2 - 4 = 0 \implies (x-1)^2 = 4$

or $\overline{[x-1]}$ are critical points (1) For $f'(x) = 0 \Rightarrow \frac{(x-1)^2}{2}$
 $\therefore x - 1 = \pm 2 \Rightarrow \therefore x = 3$

(2) <u>Note</u>: $x = 1$ is not a contract that $\frac{8}{2}$ *2 x*₋₁, $x \ne 1$
 x) is $R - \{1\} \Rightarrow J$
 4 $\frac{(x - 1)^2 - 4}{(x - 1)^2}$ *2* $2\{l\} \Rightarrow f(x)$ is continous on $R - \{l\}$
 $\frac{x-1)^2 - 4}{(x-1)^2}$
 $\frac{x-1}{(x-1)^2} = 0 \Rightarrow \therefore (x-1)^2 - 4 = 0 \Rightarrow (x-1)^2$ *Example (4)*
The domain of f (x) is $R - \{1\} \Rightarrow f(x)$ is continous on $R - \{1\}$ *e* domain
 $f'(x) = 1$ $f'(x)$ is $R - \{1\}$ =
 $\frac{4}{(x-1)^2} = \frac{(x-1)^2}{(x-1)^2}$ *1 For* $f'(x) = 1 - \frac{4}{(x-1)^2} = \frac{(x-1)^2 - 4}{(x-1)^2}$
 1) For $f'(x) = 0 \implies \frac{(x-1)^2 - 4}{(x-1)^2} = 0 \implies (x-1)^2 - 4 = 0 \implies (x-1)^2 = 4$ $\frac{(-1)^2}{x-1}$
 $\frac{(-1)^2}{x-1}$ $(x-1)^2$ $(x-1)^2$
 y $(x-1)^2$ $(x-1)^2$
 x $x-1 = \pm 2 \Rightarrow \therefore$ $(x-3)$ or $x = -1$ are critical points *2 Note : x 1 is not a critical point a* $x \neq 1$
 $I\} \Rightarrow f(x)$ is continou
 $\frac{-1}{x-1}^{2}$ \therefore y = x + x - 1
 \therefore x - 1 \therefore x + 1
 \therefore in of f (x) is R - {1} \Rightarrow f (x) is
 $= 1 - \frac{4}{(x-1)^2} = \frac{(x-1)^2 - 4}{(x-1)^2}$ x) is $R - \{1\}$ \Rightarrow $f(x)$ is conti
 $\frac{4}{-1} = \frac{(x-1)^2 - 4}{(x-1)^2}$ I } ⇒ $f(x)$ is continou
 $\frac{-I)^2 - 4}{(x - I)^2}$
 $\frac{-I)^2 - 4}{(x - I)^2} = 0$ ⇒ ∴ (x $\frac{4}{(x-l)^2} = \frac{(x-l)^2 - 4}{(x-l)^2}$
= $0 \Rightarrow \frac{(x-l)^2 - 4}{(x-l)^2} = 0 \Rightarrow : (x-l)^2 - 4 = 0 \Rightarrow (x-l)^2 = 4$ $(x-l)^2$ $(x-l)^2$
 $(l) For f'(x)=0 \Rightarrow \frac{(x-l)^2-4}{(x-l)^2}=0 \Rightarrow (x-l)^2-4=0 \Rightarrow (x-l)^2=4$
 $\therefore x-l=\pm 2 \Rightarrow \therefore \boxed{x=3}$ or $\boxed{x=-1}$ are critical points

(2) $\underline{Note:} \boxed{x=1}$ is not a critical point as it is not defined in the domain of $f(x)$

And $f(x) = \$ $(x)=0 \Rightarrow \frac{(x-1)-4}{(x-1)^2}=0 \Rightarrow (x-1)^2-4=0 \Rightarrow (x-1)^2=4$
 $2 \Rightarrow \therefore \boxed{x=3}$ or $\boxed{x=-1}$ are critical points
 $\boxed{x=1}$ is not a critical point as it is not defined in the domain of $f(x)$
 $(x)=\frac{8}{(x-1)^3} \Rightarrow \therefore f'(3)=\frac{8}{8}=1>0 \Rightarrow \therefore f(x)$ is $\begin{aligned}\n\mathbf{f} &= \pm 2 \Rightarrow \therefore \mathbf{r} \cdot \mathbf{r} \cdot \mathbf{r} = -1 \text{ and } \mathbf{r} \cdot \mathbf{r} = -1 \text{ and } \mathbf{r} \cdot \mathbf{r} = \mathbf{r} \cdot \mathbf{r} \cdot \mathbf{r} = \mathbf{r} \cdot \mathbf{$ *3* $(x-1)^2$
 \therefore $x-1=\pm 2 \Rightarrow \therefore$ $x=\overline{3}$ or $x=\overline{1}$ are critical points

(2) <u>Note</u>: $x=\overline{1}$ is not a critical point as it is not defined in the domain of $f(x)$

And $\therefore f''(x) = \frac{8}{(x-1)^3} \Rightarrow \therefore f''(3) = \frac{8}{8} = 1 > 0 \Rightarrow \therefore f$ $\boxed{x=3}$ *or* $\boxed{x=-1}$ *are c*
s not a critical point as a
 $\frac{8}{(x-1)^3} \Rightarrow$ $\therefore f''(3) = \frac{8}{8}$ And $f''(-1) = \frac{8}{8} = -1 < 0 \implies$ $\therefore f(x)$ is a local maximum point at $x = -1$ (2) <u>Note</u>: $|x=1|$ is not a critical point as it is not defined in the domain of And : $f''(x) = \frac{8}{(x-1)^3} \Rightarrow$: $f''(3) = \frac{8}{8} = 1 > 0 \Rightarrow$: $f(x)$ is a local m
And $f''(-1) = \frac{8}{8} = -1 < 0 \Rightarrow$: $f(x)$ is a local maximum point at 8
 -8 \therefore $\boxed{x=3}$ or $\boxed{x=-1}$ are critical points
 $\boxed{1}$ is not a critical point as it is not defined in the domain of $f(x)$
 $=\frac{8}{(x-1)^3} \Rightarrow$ $\therefore f''(3) = \frac{8}{8} = 1 > 0 \Rightarrow$ $\therefore f(x)$ is a local minimum point at $x =$ $x = 1$ is not a critical point as it is not defined in the domain
 $x = \frac{8}{(x-1)^3} \implies x = \int_0^x f'(3) dx = \frac{8}{8} = 1 > 0 \implies x = 1/(x)$ is a local
 $x = \frac{8}{8} = -1 < 0 \implies x = 1/(x)$ is a local maximum point at $x = 1$ *Example (5)* oint at $x = -1$
 $(x) = \begin{cases} 2x + x^2 & x \le 0 \\ 2x - x^2 & x > 0 \end{cases}$ $x = -1$
 $2x + x^2$ $x \le 0$ *Find the local maximum and the local minimum points of f (x)*
Find the local maximum and the local minimum points of f (x) at $x = -1$
= $\begin{cases} 2x + x^2 & x \le 0 \\ 2x - x^2 & x \le 0 \end{cases}$ $\begin{cases}\n2x + x^2 & x \le 0 \\
2x - x^2 & x > 0\n\end{cases}$

 $f(-1) = \frac{1}{-8} = -1 < 0 \implies$ $\therefore f(x)$ is a local maximum point at $x = -1$
 \noindent
 Example (5)
 L the local maximum and the local minimum points of $f(x) =\begin{cases} 2x + x^2 & x \leq 0 \\ 2x - x^2 & x > 0 \end{cases}$

(x) is continous at $x = 0$ as and the local minimum points of $f(x) =\begin{cases} 2x+2x-6 \\ 2x-6 \end{cases}$
= 0 as $f(0) = f(0^-) = f(0^+) = 0$ *2* $2x + x^2$ $x \le 0$
 $2x - x^2$ $x > 0$ *Answer*

Find the local maximum and the local minimum points of
$$
f(x) = \begin{cases} 2x + x^2 & x \le 0 \\ 2x - x^2 & x > 0 \end{cases}
$$

\n $\therefore f(x)$ is continuous at $x = 0$ as $f(0) = f(0^-) = f(0^+) = 0$
\n $\therefore f(x)$ is a double function, we have to get the critical points from:
\n(i) Check if : $f'(0^+) \ne f'(0^-)$ by using differentiability
\n(ii) $f'(x) = 0$
\n(i) For $f'(0^+) \colon \lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} \Rightarrow \lim_{h \to 0^+} \frac{2h + h^2 - 0}{h} \Rightarrow \lim_{h \to 0^+} \frac{h(2+h)}{h} = 2$
\nFor $f'(0^-) \colon \lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} \Rightarrow \lim_{h \to 0^+} \frac{2h - h^2 - 0}{h} \Rightarrow \lim_{h \to 0^+} \frac{h(2-h)}{h} = 2$
\nSo $\therefore f'(0^-) = f'(0^-) \Rightarrow \boxed{\therefore x = 0}$ is not a critical point
\n(ii) $\therefore f'(x) = \begin{cases} 2 + 2x & x \le 0 \\ 2 - 2x & x > 0 \end{cases}$
\nFor $x \le 0 \Rightarrow \therefore 2 + 2x = 0 \Rightarrow \boxed{x = -1 \text{ (agreed)}}$
\nFor $x > 0 \Rightarrow 2 - 2x = 0 \Rightarrow \boxed{x = 1 \text{ (agreed)}}$
\n \therefore For $x \le 0 : \therefore f''(x) = 2 \Rightarrow \therefore f''(1) = 2 > 0$
\n $\therefore x = -1$ is a local minimum of the function \Rightarrow its point is (-1, -1)
\nAnd for $x > 0 : f''(x) = -2 \Rightarrow \therefore f''(1) = -2 < 0$
\n $\therefore x = 1$ is a local maximum of the function \Rightarrow its point is (-1, -1)
\nAnd for $x > 0 : f''(x) = -2 \Rightarrow \therefore f''(1) = -2 < 0$

 \therefore $x = -1$ is a local minimum of the function \Rightarrow its point is $(-1,-1)$ $0: Y: J^-(x) = 2 \implies Y: J^-(-1) = 2 > 0$
 $\therefore x = -1$ is a local minimum of the function \Rightarrow
 $> 0: f''(x) = -2 \implies \therefore f''(1) = -2 < 0$
 $\therefore x = 1$ is a local maximum of the function

 \therefore $\mathbf{r} = \mathbf{I}$ is a local maximum of the function \Rightarrow its point is (1,1)

Example (6)

Example (6)
Prove that the function $f(x) = |x - 1|$ *has a critical point at* $x = 1$, *then show that it is a*
local minimum of the function *local minimum of the function* $\begin{aligned} \textit{ation } f(x) = |x - 1| \\\\ \textit{the function} \\\\ \textit{...} \\\\ \textit{...} \\\\ \textit{x} \geq \dots \end{aligned}$

Answer

Prove that the function
$$
f(x) = |x - 1|
$$

\nlocal minimum of the function
\n \therefore $f(x) = |x - 1| = \begin{cases} \dots & x \ge \dots \\ \dots & x < \dots \end{cases}$
\n \therefore $f(x)$ is a double function, we have
\n(i) Check if :

 $(f(x) = |x - 1| = \begin{cases} \dots & x \ge \dots \\ \dots & x < \dots \end{cases}$
 $\therefore f(x)$ is a double function, we have to get the critical points from:

(i) Check if : $\frac{f(1+h) - f(1)}{h}$ *i Check if : ... i* \therefore $f(x) = |x - 1| = \begin{cases} \dots & x \ge \dots \\ \dots & x < \dots \end{cases}$
 \therefore $f(x)$ is a double function, we have to get the critical

(i) Check if : \dots $f(1+h) - f(1) \implies \lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h} \implies \lim_{h \to 0^+} \frac{\dots}{h}$ *.......................* $\frac{h}{h}$ \Rightarrow $\lim_{h\to 0^+}$ $\frac{h}{h}$
 $\frac{h}{h}$ \Rightarrow $\lim_{h\to 0^+}$ $\frac{h}{h}$ ble function, we have to get the critical points from:
 $\lim_{h \to 0^+} \frac{f(1+h)-f(1)}{h} \Rightarrow \lim_{h \to 0^+} \frac{f(1+h)-f(1)}{$

$$
\therefore f(x) \text{ is a double function, we have to get the critical points from:}
$$
\n(i) Check if :
\n(ii) For $f'(1^+): \lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h} \Rightarrow \lim_{h \to 0^+} \frac{f(1$

Example (7)

reasing over $\big] \infty, I \big[$

the function
 $\frac{e}{f(x)}$ is linear.

(7)

(x) = $x^2 + a x + b$ has a critic

(2,1) and whether it is loce **2** *Ihus* $x = 1$ *is a local minimum of the function*
 22 *Example (7)
 22 Find the values of a and b given that the function : f (x) =* $x^2 + a x + b$ *has a critical point at* $x = 2$ *

and f (2)=1, hence determine the t* **<u>***Aote*</u> *: here we can't use second derivative because* $f(x)$ *is linear .
 Example (7)

Find the values of a and b given that the function :* $f(x) = x^2 + ax + b$ *has a critical point at and* $f(2)=1$ *, hence determine the type or local minimum* example in the same of the sa = **Example (7)**

the values of a and b given that the function : $f(x) = x^2 + ax + b$ has a critical point at x
 $f(2)=1$, hence determine the type of the point $(2,1)$ and whether it is local maximum

cal minimum
 $f(2)=1 \Rightarrow f(2)=(2)^2 +$

Answer

Find the values of a and b given that the function . *f* (*x*) = *x* + *a x* + *b* has a critical point at *x* =
and *f* (2) = 1, hence determine the type of the point (2,1) and whether it is local maximum
or local minimum

$$
\therefore f(2)=1 \Rightarrow f(2)=(2)^2 + 2a+b=1 \Rightarrow \overline{|\therefore 2a+b=-3---(1)}|
$$

The function has a critical point at *x* = 2 means that we must find *f*'(2) = 0

$$
f'(x)=2x+a \Rightarrow \therefore f'(2)=0 \Rightarrow 4+a=0 \Rightarrow \overline{a=-4}|
$$

Substitute in (1): 2(-4)+*b* = -3 ⇒ $\overline{|\therefore b=-3+8=5|}$
Then the function is : *f* (*x*) = *x*² - 4*x* + 5
∴ *f*''(*x*) = 2 ⇒ then at *x* = 2 : *f*''(2) = 2>0 ⇒ then *f* (*x*) has a local minimum at *x* = 2

Case (3): Absolute maximum & minimum

 Steps

(1) Differentiate the fun

(2) Solve the equation f

(3) Take the critical point

(4) Substitute in the ori (1) Differentiate the fun
(2) Solve the equation f
(3) Take the critical poi
(4) Substitute in the ori
(5) Choose the highest *(1) Differentiate the function* $f(x)$ and get $f'(x)$.

2) Differentiate the function $f(x)$ and $get f'(x)$.
 2) Solve the equation $f'(x) = 0$ to find the values of the critical points. *3 4 Take the function* $f(x)$ *and get* $f'(x)$.
 3 Take the equation $f'(x) = 0$ *to find the values of the critical 3) Take the critical points which are inside the given interval. 4 Substitute ine fund 2)*
4 Solve the equation f
4 Substitute in the ori
4 Substitute in the ori =

ginal function by using the critical points.

(1) Differential emergination $f'(x) = 0$ to find the values of the critical points.

(3) Take the critical points which are inside the given interval.

(4) Substitute in the original function by using the critical points.

Example (1)

(3) Take the critical points which are inside the given interval.

(4) Substitute in the original function by using the critical points.

(5) Choose the highest value (maximum) and choose the lowest value (minimum).
 Exa

Answer (*noose the highest value (maximum) and choose the lowest value (minimum).*
 Fxample (1)
 f (*x*) *is continous on* [1,3] $\Rightarrow f'(x) = 12 - 3x^2 = 3(2-x)(2+x)$
 f $f'(x) = 0 \Rightarrow 3(2-x)(2+x) = 0 \Rightarrow x = 2$ *or* $x = 2$ **Example (1)**
the absolute maximum and the absolute minimum of .
(x) = 0 \Rightarrow 3(2-x)(2+x)=0 \Rightarrow $x=2$ o
 $2 \ne 11$ 31 \Rightarrow the only critical point at $\boxed{x=2}$ *f f (x) is continous on* $\begin{bmatrix} 1,3 \end{bmatrix} \Rightarrow f'(x) = 1$
 Answer
 for f'(x) *is continous on* $\begin{bmatrix} 1,3 \end{bmatrix} \Rightarrow f'(x) = 12 - 3x^2 = 3(2-x)(2+x)$
 for f'(x)=0 $\Rightarrow 3(2-x)(2+x) = 0 \Rightarrow \boxed{x=2}$ *or* $\boxed{x=-2}$
 But $-2 \notin [1,3] \Rightarrow$ $f'(x)$ is continous on [1,3] $\Rightarrow f'(x) = 12 - 3x^2 = 3(2-x)(2+x)$

for $f'(x) = 0 \Rightarrow 3(2-x)(2+x) = 0 \Rightarrow \boxed{x=2}$ or $\boxed{x=-2}$

But -2 ∉ [1,3] \Rightarrow ∴ the only critical point at $\boxed{x=2}$

∴ $f(2) = 1 + 24 - 8 = 17$ and $f(1) = 1 + 12 - 1 = 12$ and *2 z f* (*x*) *is continous on* [1,3] $\Rightarrow f'(x) = 12 - 3x^2 = 3(2-x)(2+x)$ $B(f(x))$ is continuous on $[1,3] \Rightarrow f'(x) = 12 - 3x^2 = 3(2 - 1)$
 $\text{for } f'(x) = 0 \Rightarrow 3(2-x)(2+x) = 0 \Rightarrow \boxed{x=2}$ or $But - 2 \notin [1,3] \Rightarrow$ \therefore the only critical point at $\boxed{x=2}$ *Thus the absolute maximum is 17 at* $x =$ absolute maximum and the absolute minimum of : $f(x) = 1 + 12x - x^3$ w

continous on $[1,3] \Rightarrow f'(x) = 12 - 3x^2 = 3(2-x)(2+x)$
 $= 0 \Rightarrow 3(2-x)(2+x) = 0 \Rightarrow \boxed{x=2}$ or $\boxed{x=-2}$ is continuous on $[1,3] \Rightarrow f'(x) = 12 - 3x^2 = 3(2-x)$
 $x) = 0 \Rightarrow 3(2-x)(2+x) = 0 \Rightarrow \boxed{x=2}$
 $\in [1,3] \Rightarrow \therefore$ the only critical point at $\boxed{x=1}$ \Rightarrow \therefore the only critics
 $-8 = 17$ and $f(1) = 1 + 1$
 e maximum is 17 at $x = 2$
 e minimum is 10, at $x = 3$ *But -2* \notin [1,3] \Rightarrow \therefore the only critic
 \therefore $f(2)=1+24-8=17$ and $f(1)=1+26$
 And the absolute maximum is 10 at x = 3
 And the absolute minimum is 10 at x = 3 And the absolute minimum is 10 at $x-3$

Example (2)

 $(x)=x^3-3x^2-9x+1$ *Example (2)
<i>Determine the absolute maximum and minimum of the function : f (x)* = $x^3 - 3x^2 - 9x + 1$
on [-2,4]
Answer on $[-2, 4]$ $= x^3 - 3x^2 - 9x + 1$ Example the absolute max
 $[-2, 4]$

(x) is continous on R
 $[x] = 3x^2 - 6x - 9 = 3(x - 1)$

Answer

Determine the absolute maximum and minimum of the function :
$$
f(x) = x^3 - 3x^2 - 9x + 1
$$

\non $[-2,4]$
\n $\therefore f(x)$ is continuous on R
\n $f'(x) = 3x^2 - 6x - 9 = 3(x-3)(x+1) \Rightarrow$ for $f'(x) = 0$
\n $\therefore \quad \boxed{x=3}$ or $\boxed{x=-1}$ are the Critical points and $\in [-2,4]$
\n $f(-2) = (-2)^3 - 3(-2)^2 - 9(-2) + 1 = -1$ $f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 1 = 6$
\n $f(3) = (3)^3 - 3(3)^2 - 9(3) + 1 = -26$ $f(4) = (4)^3 - 3(4)^2 - 9(4) + 1 = -19$
\nThen the absolute maximum point is $(-1,6)$ And the absolute minimum point $(3,-26)$
\nExample (3)
\nFind the absolute maximum and absolute minimum of $f(x) = \frac{x}{x-1}$ in $[2,4]$

Example (3)

Example

ind the absolute maximum and absolute minin
 $f(x)$ is continous on [2,4] Find the absolute maximum and absolute minimum of $f(x) = \frac{x}{x-1}$ in
 $\frac{\text{Answer}}{x \cdot f(x)}$
 $(f(x) \text{ is continuous on } [2,4]$
 $(f(x) \text{ is continuous on } [2,4])$
 $(f(x) = \frac{-1}{(x-1)^2} \Rightarrow \therefore \text{ when } f'(x) = 0 \Rightarrow -1 = 0 \text{ (no real solution)}$
 $(2) \ f'(x) \text{ is undefined when denominator } = 0 \Rightarrow (x-1)^2 = 0 \Rightarrow \exists x = 1$ *Find the absolute maximum point is* $(-1,6)$ *And the absolute minimum point* $(3,-2)$
Example (3)
Find the absolute maximum and absolute minimum of $f(x) = \frac{x}{x-1}$ *in* [2,4 $\frac{x}{x-1}$ = − *Answer*

Answer
 $f(x)$ is continous on [2,4]

(1) $\therefore f'(x) = \frac{-1}{(x-1)^2} \Rightarrow \therefore$ when $f'(x) = 0 \Rightarrow -1 = 0$ (no real solution)

(2) $f'(x)$ is undefined when denominator $= 0 \Rightarrow (x-1)^2 = 0 \Rightarrow \therefore x = 1$ (refused)
 \therefore There is no critical point *2 f* $f(x)$ *is continous on* [2,4]
 1) $\therefore f'(x) = \frac{-1}{(x-1)^2} \Rightarrow \therefore$ when $f'(x)=0 \Rightarrow -1=0$ (no real solution $\frac{-I}{x - I}$ *2 f ' x is continous on* [2,4]
 2) *z <i>f '*(*x*) = $\frac{-1}{(x-1)^2}$ \Rightarrow *z when f'*(*x*) = 0 \Rightarrow -1 = 0 (*no real solution*)
 2) *f'*(*x*) *is undefined when denominator* = 0 \Rightarrow $(x-1)^2 = 0$ \Rightarrow $\frac{1}{x}$ *x* = $f'(x)$ is continous on [2,4]
 $\therefore f'(x) = \frac{-1}{(x-1)^2} \Rightarrow \therefore$ when $f'(x) = 0 \Rightarrow -1 = 0$ (no real solu $= 0 \Rightarrow -1 = 0$ (no real solution)
 $= 0 \Rightarrow (x-1)^2 = 0 \Rightarrow \boxed{\therefore x = 1 \text{ (refused)}}$ $f'(x) = \frac{1}{(x-1)^2} \implies$ \therefore when $f'(x) = 0 \implies -1 = 0$
 $f'(x)$ is undefined when denominator = 0 \implies $(x-1)$
 \therefore There is no critical points in this function
 \therefore $f(2) = \frac{2}{2-1} = 2$ and $f(4) = \frac{4}{4-1} = \frac{4}{3}$

(2) *f'*(*x*) is undefined when denominator = 0 ⇒ $(x-1)^2$

∴ There is no critical points in this function

∴ $f(2) = \frac{2}{2-1} = 2$ and $f(4) = \frac{4}{4-1} = \frac{4}{3}$ inator = 0 \Rightarrow $(x - 1)^2$ =
this function
= $\frac{4}{4-1} = \frac{4}{3}$

 There is no critical points in this function

$$
\therefore f'(x) = \frac{}{(x-1)^2} \Rightarrow \therefore \text{ when } f'(x) = 0
$$

$$
f'(x) \text{ is undefined when denominator} = 0
$$

$$
\therefore \text{ There is no critical points in this function}
$$

$$
\therefore f(2) = \frac{2}{2-1} = 2 \text{ and } f(4) = \frac{4}{4-1} = \frac{4}{3}
$$

4 ∴ There is no critical points in this function

∴ $f(2) = \frac{2}{2 - 1} = 2$ and $f(4) = \frac{4}{4 - 1} = \frac{4}{3}$

∴ The absolute maximum is 2 at $x = 2$ and the absolute minimum is $\frac{4}{3}$ at $x = 4$ \therefore The absolute maximum is 2 at $x = 2$ and the absolute minimum is $\frac{1}{x}$ at $x =$

Example (4)

∴ $f(2) = \frac{2}{2-1} = 2$ and $f(4) = \frac{4}{4-1} = \frac{4}{3}$

∴ The absolute maximum is 2 at $x = 2$ and the absolute minimum is $\frac{4}{3}$ at $x = 4$

Example (4)

Determine the absolute maximum and minimum of the function : $f(x) = x e$ *Answer f* (*x*) *is continous on* $[0,2)$

The absolute maximum is 2 at $x = 2$ and the

Example

Example

Example

Example
 (x) is continous on [0,2]
 $(x) = -x e^{-x} + e^{-x} \implies for f'(x) = 0 \implies -xe^{-x}$ **Example (4)**

termine the absolute maximum and minimum of the function : $f(x) = x$
 $f(x)$ is continous on [0,2]
 $(x) = -x e^{-x} + e^{-x} \Rightarrow$ for $f'(x) = 0 \Rightarrow -x e^{-x} + e^{-x} = 0$ (divide by e^{-x})
 $x + 1 = 0 \Rightarrow x = \sqrt{x-1}$ is the Critical poin function : $f(x) = x e^{-x}$ ove
 $= 0$ (divide by e^{-x})
 $[0,2]$
 $= (2)e^{-2} \approx 0.27$ $f(x)$ is continous on [0,2]
 $f(x) = -x e^{-x} + e^{-x} \implies$ for $f'(x) = 0 \implies -x e^{-x} + e^{-x} = 0$ (divide by e^{-x})
 $-x + 1 = 0 \implies \therefore$ $\boxed{x = 1}$ is the Critical point and $\in [0,2]$
 $(0) = 0$ $f(1) = e^{-1} \approx 0.37$ $f(2) = (2)e^{-2} \approx 0.27$

then th $0 \Rightarrow -x e^{-x} + e^{-x} = 0$ (dividentical point and $\in [0,2]$
 $f(2) = (2)e^{-2} \approx (1,0.37)$ and the absolute minority *Position on* $[0,2]$
 $\begin{aligned}\n\frac{\text{Answer}}{2} \\
\frac{1}{2}x + e^{-x} \implies \text{for } f'(x) = 0 \implies -xe^{-x} + e^{-x} = 0 \text{ (divide by } e^{-x}\n\end{aligned}$ *the Critical point and* $\in [0,2]$
⁻¹ \simeq 0.37 $f(2)=(2)e^{-2}$ *Determine the absolute maximum and minimum of the function : f (x)*
 f '(x) is continous on [0,2]
 f '(x) = -*x e^{-x}* + e^{-x} \Rightarrow for f '(x) = 0 \Rightarrow -*x e^{-x}* + e^{-x} =0 (divide by e *f*(*x*) is continous on [0,2]
 $(x) = -xe^{-x} + e^{-x} \implies for \ f'(x) = 0 \implies -xe^{-x} + e^{-x} = 0$
 $-x + 1 = 0 \implies \therefore \boxed{x = 1}$ is the Critical point and $\in [0, 2]$ $f'(x) = -x e^{-x} + e^{-x} \implies for \ f'(x) = 0 \implies -x e^{-x} + e^{-x} = 0$ (divide by
 $\therefore -x + 1 = 0 \implies \therefore \boxed{x = 1}$ is the Critical point and $\in [0,2]$
 $f(0) = 0$ $f(1) = e^{-1} \approx 0.37$ $f(2) = (2)e^{-2} \approx 0.27$ $f(0)=0$ $f(1)=e^{-1} \approx 0.37$ $f(2)=(2)e^{-2} \approx 0.27$
Then the absolute maximum point is $(1,0.37)$ and the absolute minimum point $(0,0)$ mine the absolute maximum and minimum of the function : $f(x) = \frac{Answer}{4}$

:) is continous on [0,2]
 $= -x e^{-x} + e^{-x} \Rightarrow for \ f'(x) = 0 \Rightarrow -x e^{-x} + e^{-x} = 0$ (divide by e : $f(x)$ is continous on [0,2]
 $f'(x) = -x e^{-x} + e^{-x} \implies for \ f'(x) = 0$
 $\therefore -x + 1 = 0 \implies \therefore \boxed{x = 1}$ is the Critice $f(1) = e^{-1} \simeq 0.37$ \in *ute by e^{-x}* (*divide by e^{-x})*
 ute minimum point (0,0)

Case (4): Concavity and point of inflection

flection point occurs
 For polynomial function

(**i**) $f(x)$ is continous, d

(**ii**) The sign of curvature
 For fractional function *Inflection point occurs in one of the following cases* $\overline{(i)}$ *For polynomial functions***
** $\overline{(i)}$ *f***(***x***)** *is continous***,** *differentiable and**f* $''(x) = 0$ **
(***ii***)** *The sign of curvature changes from concave up to do*

1 For polynomial functions

=

(1) **For polynomial functions**

(i) $f(x)$ is continous, differentiable and $f''(x)=0$

(ii) The sign of curvature changes from concave up to down or vice versa

(2) **For fractional functions** (derivatives)

(i) $f(x)$ is con f $f''(x) = 0$
we up to down or vice ver.
 $(x) = 0$
es of the denominator of t.

- *(i)* $f(x)$ is continous, differentiable and
 (ii) The sign of curvature changes from co

2) **For fractional functions** (derivatives)
 (i) $f(x)$ is continous, differentiable and
 (ii) $f'(x)$ doesn't exist except the *(i)* $f(x)$ is c
 (ii) The sign
 (2) For fractio
 (i) $f(x)$ is c
 (ii) $f'(x)$ de *For fractional functions*
	- *ontinous*, *differentiable and* $f(x) =$
of curvature changes from concave up t
mal functions (derivatives)
ontinous, *differentiable and* $f''(x) = 0$ =
- (*i*) $f(x)$ is continous, differentiable and $f''(x) = 0$

(*ii*) The sign of curvature changes from concave up to down or vice versa

(2) **For fractional functions** (derivatives)

(*i*) $f(x)$ is continous, differentiable a For fractional functions (derivatives)

(i) $f(x)$ is continous, differentiable and $f''(x)=0$

(ii) $f'(x)$ doesn't exist except the set of zeroes of the denominator of the original

For double functions

(i) $f(x)$ must be c $\begin{aligned} & \text{intermin} \ & \text{intermin}$ $f(x) = 0$
c of the denominator of the original j
 $\[f'(a^-) = f'(a^+) \]$

(*ii*) The sign of curvate

(2) **For fractional function**

(*i*) $f(x)$ is continous

(*ii*) $f'(x)$ doesn't exis

(3) **For double function** *3 For double functions*

(i) f '*(x) is continous*, *afferentiable and f* $(x) = 0$
(ii) $f'(x)$ *doesn't exist except the set of zeroes of the den*
(3) **For double functions**
(i) $f(x)$ must be continous and differentiable $\left[f'(a^-) =$
 = (*i*) $f'(x)$ is continuous, any perentiable and $f'(x) = 0$

(*ii*) $f'(x)$ doesn't exist except the set of zeroes of the denomin
 For double functions

(*i*) $f(x)$ must be continuous and differentiable $\left[f'(a^-) = f'(a^-) \right]$ (3) For double functions

(i) $f(x)$ must be continous and differe

(ii) $f'(x) = 0$ or $f'(x)$ doesn't exist +

(ii) $f'(x)=0$ or $f'(x)$ doesn't exist

Steps

- **Steps**

(1) Find $f''(x)$, then solve $f''(x)=0$.

(2) Determine the sign of $f''(x)$ before and after each. **5teps**
 1) Find f''(x), then solve f''(x)=0.

2) Determine the sign of f''(x) hefor =
- **Steps**

(1) Find $f''(x)$, then solve $f''(x)=0$.

(2) Determine the sign of $f''(x)$ before and after eac

(i) $f''(x)$ changes its sign before and after this po **(a)** $\begin{aligned} \textbf{Steps} \\ \textbf{1) Find } f''(x), \text{ then solve } f''(x) = 0. \end{aligned}$

2) Determine the sign of $f''(x)$ before and after each point if :

(i) $f''(x)$ changes its sign before and after this point, then this po

an inflection point *(1) Find f* "(*x*), then solve $f''(x) = 0$.
 (2) Determine the sign of f "(*x) before and after each point....... if :*
 (i) f "(*x) changes its sign before and after this point, then this point is*
 an inflection po an inflection point. (1) Find f "(*x*), then solve $f''(x) = 0$.
 (2) Determine the sign of f "(*x) before and after each point....... if :*
 (i) f "(*x) changes its sign before and after this point, then this point is not an inflection*
	- *an inflection point .*

Example (1)

 $|x=3|$ $f''(0) = -ve$ $f''(3) = +ve$ *Concave down over - ,2 Concave up over 2,* $\begin{array}{c|c|c|c} \hline \text{Convex up} & & & & \text{convex up} \ \hline \text{Convex down over }]-\infty,2[& & & ++++++++++++++++++ \\\hline \text{So, since there is a Concave down then Concave up , this means that } x=2 \text{ is an inflection point, then substitute in the original function : } f(2)=(2)^3-6(2)^2+2+1=-13 \\\hline \text{Then the inflection point is } (2,-13) & & & \text{if } x=2 \text{ and } y=0. \end{array}$ *point , then substitute in the original function* $point$ *is* $(2,-13)$
point in the inflection point is $(2,-13)$
point if the substitute in the original function : f $(2,-13)$ So, since there is a **Concave down** then **Concave up**, this means that $x = 2$ is an inflection $\begin{array}{c|c}\n\textbf{C}urvature changes & \textbf{Example (1)}\\ \n\textbf{D} is cuss the concavity of: \ \ f(x) = \ x^3 - 6x^2 + x + 1 & \textbf{A} \end{array}$ *Answer Curvature changes*
Exa
f (x) is continous and differentiable on \overline{R}
 $f(x)$ *is continous and differentiable on* \overline{R}
 $f(x) = 3x^2 - 12x + 1 \implies f''(x) = 6x - 12$ (*Discuss the concavity of :* $f(x) = x^3 - 6x^2 + x + 1$
 $\therefore f(x)$ *is continous and differentiable on* R
 $f'(x) = 3x^2 - 12x + 1 \implies f''(x) = 6x - 12 \implies for \ f''(x) = 0 \implies 6x - 12 = 0 \implies \therefore x = 2$

To check that this point is an inflection or n *To check that this point is an inflection or not :* $f'(x)$ is continous and differentiable on \overline{R}
 $f'(x) = 3x^2 - 12x + 1 \implies f''(x) = 6x - 12 \implies$ for \overline{R}
 To check that this point is an inflection or not :
 Take point before $\boxed{x = 2}$ *and another point after* $\boxed{x =$ **Example** (1)

s the concavity of : $f(x) = x^3 - 6x^2 + x + 1$
 $\frac{Answer}{x}$

c) is continous and differentiable on R
 $= 3x^2 - 12x + 1 \Rightarrow f''(x) = 6x - 12 \Rightarrow for f''(x) = 0 \Rightarrow 6x - 12 = 0 \Rightarrow \therefore x = 2$

ook that this point is an inflection or not: and differentiable on R
 $\Rightarrow f''(x) = 6x - 12 \Rightarrow for f''(x) = 0 \Rightarrow 6$

int is an inflection or not :
 $\equiv 2$ and another point after $x = 2$ *- + + + + + + + + + + + + + + + + + +*

point, then substitute in the original function : $f(2) = (2)^3 - 6(2)^2 + 2 + 1 = -13$ *Then the inflection point is (2.-13)*

Example (2)

Example (2)
Consider the function $f(x) = \frac{1}{20}x^5 - \frac{1}{12}x^4 + x + 6$, find the inflection point of $f(x)$ if exists
and describe the concavity of f . *and describe the concavity of f .* **Example** (2)
= $\frac{1}{20}x^5 - \frac{1}{12}x^4 + x + 6$, find the inflec sider the function $f(x) =$
describe the concavity of $f(x)$
 (x) is continous and difference $f(x) = \frac{1}{2}x^4 - \frac{1}{2}x^3 + 1 \Rightarrow f''(x)$

Answer

Example (3)

and convex downwards and also find the points of inflection if exists . Final state

rmine the intervals over w

convex downwards and als

(x) is continous and differ
 $f(x) = 2x \implies f''(x) = 2 > 0$ () () () *f x is continous and differentiable on R* the the thermal state which the earte of the
novex downwards and also find the points of
 Δ nswe
 \Rightarrow \Rightarrow $f''(x) = 2 > 0 \Rightarrow$ then the function
 \therefore the gurns does not change its conservity at

Answer

and convex downwards and also find the points of inflection if exists .
 $\frac{\text{Answer}}{\text{4.1.}}$
 $f'(x) = 2x \implies f''(x) = 2 > 0 \implies$ then the function is concave upward (convex down).

And \therefore the curve does not change its concavity *Answer*
 $f'(x) = 2x \implies f''(x) = 2 > 0 \implies$ then the function is concave upward (convex down).

And : the curve does not change its concavity at any point, so there is no inflection points for this function. *for this function .*

Example (4)

(4)
 $(x) =\begin{cases} x^2 - 4 & x < -2 \\ x^3 - 3x + 2 & x \ge -2 \end{cases}$ *2 3 Example (4)*
Determine the intervals of convexity of the function f $(x) = \begin{cases} x^2 - 4 & x < -2 \\ x^3 - 3x + 2 & x \ge -2 \end{cases}$ *, then find the* $x^2 - 4$ $x < -2$
 $x^3 - 3x + 2$ $x \ge -2$ $=\begin{cases} x^2-4 & x < -2 \\ x^3-3x+2 & x > -2 \end{cases}$, then find then $\begin{cases} x^2-4 & x < -2 \\ x^3-3x+2 & x \ge -2 \end{cases}$, then find th

inflection point and the equation of tangent if exists .

Example (4)
\n*Determine the intervals of convexity of the function*
$$
f(x) = \begin{cases} x^2 - 4 & x < -2 \\ x^3 - 3x + 2 & x \ge -2 \end{cases}
$$
, then find the
\n $inflection point and the equation of tangent if exists.$
\n $\therefore f(x)$ is continuous at $x = -2$ as $f(-2) = f(-2^{-1}) = 0$
\n $\therefore f(x)$ is a double function, we have to get the critical points from:
\n(i) Check if : $f'(2^{-1}) = f'(2^{-1})$ by using differentiability (ii) $f''(x) = 0$
\n(i) For $f'(2^{-1})$: $\lim_{h \to 0^{-}} \frac{f(2+h)^2 - f(2)}{h} \Rightarrow \lim_{h \to 0^{-}} \frac{(-2+h)^2 - 4 - [(-2)^2 - 4]}{h} = -4$
\nFor $f'(2^{-1})$: $\lim_{h \to 0^{-}} \frac{f(2+h)^3 - 3(-2+h) + 2 - [(-2)^2 - 3(-2)+2]}{h} = 9$
\n $\therefore f'(2^{-1}) \neq f'(2^{-1}) \Rightarrow then \boxed{x=2}$ is not an inflection point
\n(ii) $\therefore f'(x) = \begin{cases} 2x & x < -2 \\ 3x^2 - 3 & x \ge 2 \end{cases}$ $f''(x) = \begin{cases} 2 & x < -2 \\ 6x & x \ge 2 \end{cases}$
\nFor $x < -2$: $2 = 0 \Rightarrow$ Can't be
\nFor $x \ge 2$: $6x = 0 \Rightarrow \frac{1}{x} \cdot x = 0$ (agreed) is an inflection point
\n $f''(-3) = +ve$
\n $\frac{x = -2}{0}$ $f''(-1) = -ve$
\n $\frac{x = 0}{0}$ $f''(1) = +ve$
\n $f''(-3) = +ve$
\n $\frac{x = -2}{0}$ $f'''(-1) = -ve$
\n $\frac{x = 0}{0}$ $f'''(1) = +ve$
\n $f'''(-3) = +ve$
\n $\frac{x = -2}{0}$ $f'''(-1) = -ve$ <

Example (5)

 $(x) = \begin{cases} 2x^2 + ax + b \\ 3x - x^2 \end{cases}$ If the function $f(x)$
(1) Find the constant $f(x)$ *If the function f* (*x*) = $\begin{cases} 2x^2 \\ 3x \end{cases}$
(*1*) *Find the constants a ar* (2) *Determine the interval 2x² + ax + b* $x \ge 1$ *

<i>2x*² + ax + b $x \ge 1$ *is differentiable on R* $x < 1$ $2x^2 + ax + b$ $x \ge 1$
 $3x - x^2$ $x < 1$ *f* the function $f(x) = \begin{cases} 2x^2 + a \\ 3x - x^2 \end{cases}$
 1) *Find the constants a and b 2 Determine the intervals of convexity up and convexity down .*
 2 Determine the intervals of convexity up and convexity down . **Example** (.
= $\begin{cases} 2x^2 + ax + b & x \ge 1 \\ 2x^2 & \text{is different} \end{cases}$ **Example** (
 $\begin{cases} 2x^2 + ax + b & x \ge 1 \\ 3x - x^2 & x < 1 \end{cases}$ is different

 (1) Find the constants a and b

- *3 Find the constants a and b*
3 Potermine the intervals of convexity up an
3 Find the inflection point and the equation
- *(3) Find the inflection point and the equation of tangent if exist.*

Answer

\n- (1) Find the constants *a* and *b*
\n- (2) Determine the intervals of convexity up and convexity down.
\n- (3) Find the inflection point and the equation of tangent if exist.
\n- $$
x \leq 1
$$
\n- $x \leq 1$
\n- $x \leq 1$
\n- $f(x)$ is differentiable at $x = 1 \implies \therefore f(x)$ is continuous at $x = 1$
\n- $f(1) = f(1^+) = 2 + a + b - -(-1)$ and $f(1^-) = \lim 3x - x^2$
\n

\n- (1) Find the constants *a* and *b*
\n- (2) Determine the intervals of convexity up and convexity down.
\n- (3) Find the inflection point and the equation of tangent if exist.
\n- ∴
$$
f(x)
$$
 is differentiable at $x = 1 \Rightarrow \therefore f(x)$ is continuous at $x = 1$
\n- ∴ $f(1) = f(1^+) = 2 + a + b - -(-1)$ and $f(1^-) = \lim_{x \to 1^-} 3x - x^2 = 2 - -(-2)$
\n- ∴ From (1) and (2): $2 + a + b = 2 \Rightarrow \boxed{\therefore a + b = 0 - -(-3)}$
\n- ∴ $f(x)$ is a double function, we have to get the critical points from:
\n- ∴ $f(x)$ is differentiable at $x = 1 \Rightarrow f'(1^+) = f'(1^-)$
\n- ∴ $f(1+h) - f(1)$ and $f(1+h) - (1+h)^2 -[3(1) - (1)^2]$
\n

- *f f* (*x*) *is a double function, we have to get the critical points from* $f(x)$ *is a double function, we have to get the critical points from* $f(x)$ *is differentiable at* $x = 1 \implies f'(1^+) = f'(1^-)$ $\begin{aligned} \therefore \quad & a + b = b = -(-5) \\ \therefore & \therefore \text{ the critical points } f \cdot \\ & \therefore \text{ the critical points } f \cdot \\ & \therefore \end{aligned}$
- *rom:*

$$
\therefore f(x) \text{ is differentiable at } x = 1 \implies f'\left(1^+\right) = f'\left(1^-\right)
$$

$$
f(1) = f(1^{-1}) = 2 + a + b - - - (1)
$$
 and $f(1^{-1}) = \lim_{x \to 1^{-}} 3x - x^{2} = 2 - - - (2)$
\n
$$
f(x) \text{ is a double function, we have to get the critical points from:}
$$

\n
$$
f'(x) \text{ is a double function, we have to get the critical points from:}
$$

\n
$$
f'(x) \text{ is differentiable at } x = 1 \implies f'(1^{+}) = f'(1^{-})
$$

\n
$$
(i) \text{For } f'(1^{-}) : \lim_{h \to 0^{+}} \frac{f(1+h) - f(1)}{h} \implies \lim_{h \to 0^{+}} \frac{3(1+h) - (1+h)^{2} - [3(1) - (1)^{2}]}{h} = 1
$$

\n
$$
\text{For } f'(1^{+}) : \lim_{h \to 0^{+}} \frac{2(1+h)^{2} + a(1+h) + b - [2 + a + b]}{h} = 4 + a
$$

\n
$$
\therefore 4 + a = 1 \implies \text{then } \boxed{a = -3} \implies \text{from (3)} \quad \therefore \text{ } b = 3 \implies \therefore f(x) = \begin{cases} 2x^{2} - 3x + 3 & x \ge 1 \\ 3x - x^{2} & x < 1 \end{cases}
$$

\n
$$
(ii) \therefore f'(x) = \begin{cases} 4x - 3 & x \ge 1 \\ 3 - 2x & x < 1 \end{cases} \implies f''(x) = \begin{cases} 4 & x \ge 1 \\ -2 & x < 1 \end{cases}
$$

For
$$
f'(1^+): \lim_{h\to 0^+} \frac{2(x+h)-x}{h} \frac{2(x+h)-x}{h} = 4+a
$$

\n $\therefore 4+a=1 \Rightarrow \text{ then } \boxed{a=-3} \Rightarrow \text{ from (3)} \quad \boxed{\therefore b=3} \Rightarrow \therefore f(x) = \begin{cases} 2x^2-3x+3 & x \ge 1 \\ 3x-x^2 & x < 1 \end{cases}$
\n $f'(x) = \begin{cases} 4x-3 & x \ge 1 \\ 3-2x & x < 1 \end{cases} \Rightarrow f''(x) = \begin{cases} 4 & x \ge 1 \\ -2 & x < 1 \end{cases}$

ii f ' x f '' x 3 2x x 1 -2 x 1 For x 1: -2 0 Can' t be For x 1 : 4 0 Can' t be ⁼ = − = = () () *The only inflection point is at x 1 : f 1 2 the inflection point is 1 2* = = *f '' 0 ve* () = − *f '' 2 ve* () = + *Convex up Convex down x 1* =

Concave down over - ,1 Concave up over 1, - + + + + + + + + + + + + + + + + + +

 $\begin{array}{|l|l|}\n\hline\n\text{over }]-\infty, I[\quad \text{if} \quad \text{C} \text{d} \\\n\text{over }]-\infty, I[\quad \text{if} \quad \text{C} \text{d} \\\n\text{at } \boxed{x=1} : \quad \text{if} \quad (I) = 2 \implies \quad \text{if} \quad \text{on} \text{of tangent at this point} \\\n\text{(1, 2)} \quad \text{if} \quad \text{d} \text{d} \\\n\end{array}$ *, Then we can find the equation of tangent at this point Concave down over*
ie only inflection point is at \overline{x}
ien we can find the equation of
The point of tangency is $(1, 2)$
 $(x-3, y>1)$

The only inflection point is at
$$
x = 1
$$
 : $f(1) = 2 \implies$ \therefore the inflection point is $(1, 1)$.
\nThen we can find the equation of tangent at this point
\n \therefore The point of tangency is $(1, 2)$
\nAnd $f'(x) =\begin{cases} 4x-3 & x \ge 1 \\ 3-2x & x < 1 \end{cases} \implies \therefore$ at $1 = 0 \implies \frac{dy}{dx} = 1$
\nThen the equation of tangent line : $y - 2 = (x - 1) \implies \frac{dy}{dx} = x - 1 = 0$